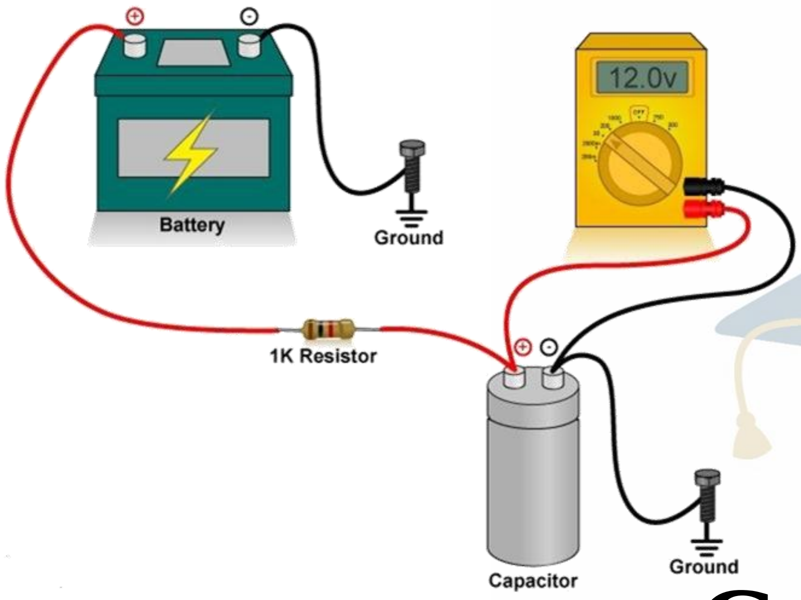


Grade 12 LS – Physics

Chapter 10 -A

Capacitor with a L.F.G of square signal

Prepared & Presented by: Mr. Mohamad Seif

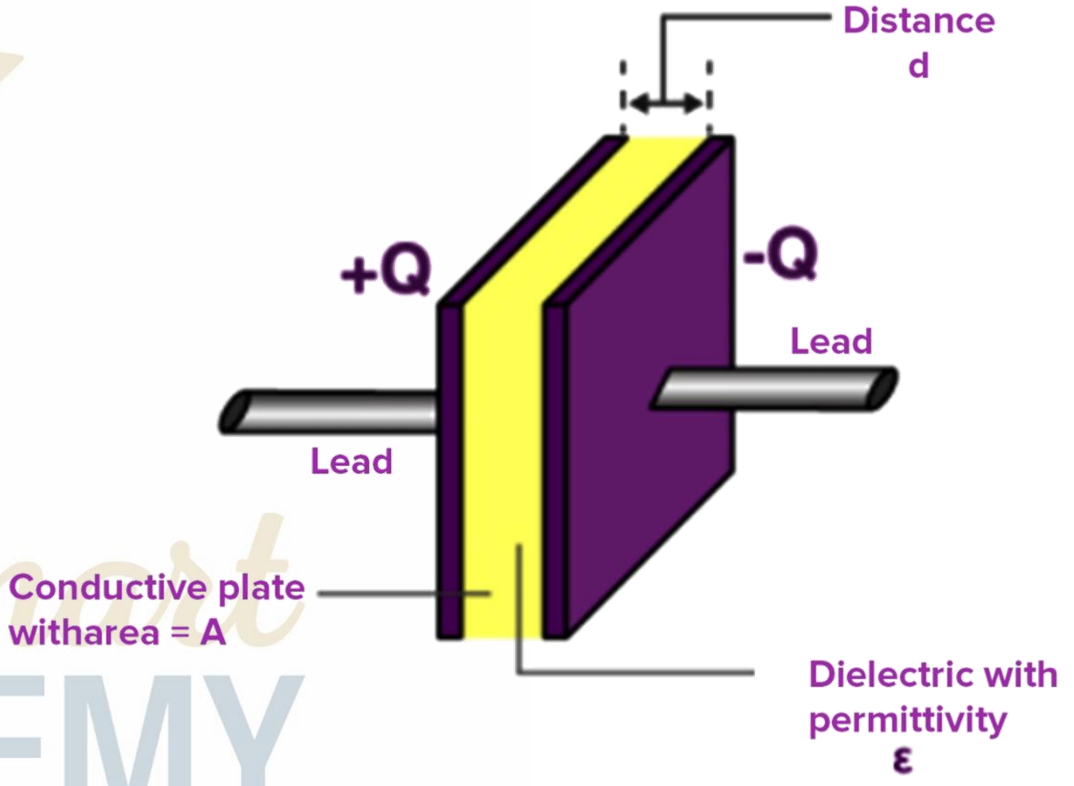
OBJECTIVES

1. Definition of a capacitor.
2. Capacitance of a capacitor.
3. Energy stored in a capacitor.

Definition of capacitor

What is capacitor?

A capacitor is an electric device formed of two conducting parallel plates (armatures) separated by an insulator called dielectric which can be vacuum, air, glass, ceramic.

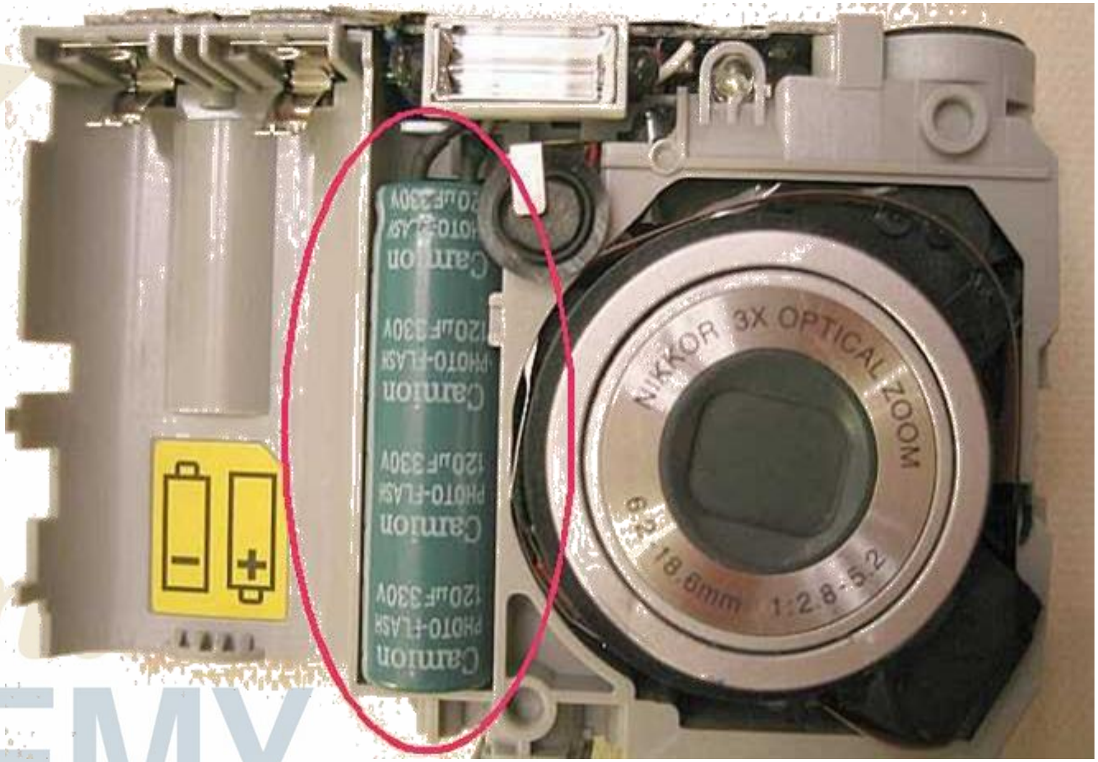


Definition of capacitor

Where is the capacitor used?

The capacitor is found and used in many electric devices such as:

- Computers
- Camera flash
- Alarms …



Definition of capacitor

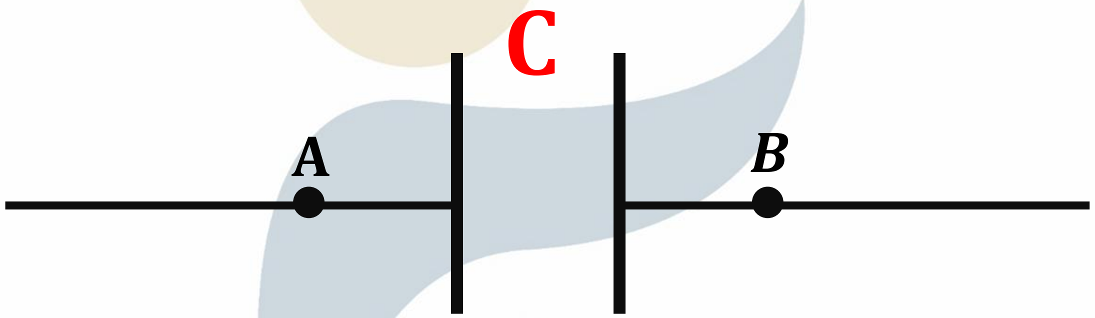
What is the main function of the capacitor?

The capacitor is manufactured to store electric energy and to return it to the circuit whenever required.

Definition of capacitor

The representation of a capacitor?

The capacitor is represented by the symbol



Symbol of Capacitor

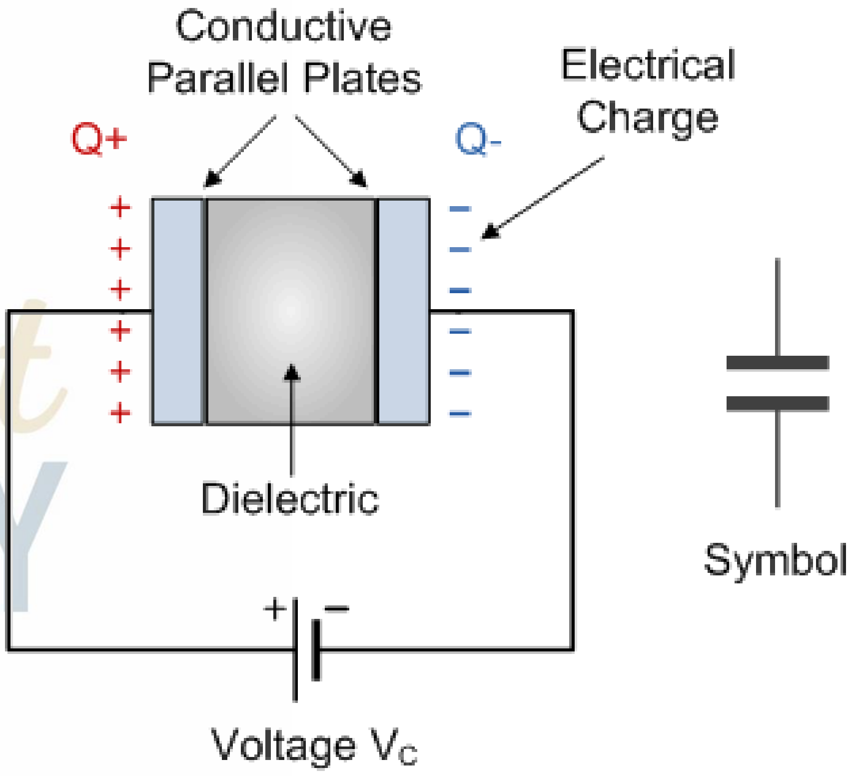
When the plates are not charged, we say that the capacitor is <u>neutral.</u>

Capacitance of a capacitor

What is capacitance?

Capacitance is the electrical property of a capacitor and is the measure of a capacitor's ability to store an electrical charge onto its two plates.

The SI unit of capacitance being the Farad (F) named relative to the British physicist Michael Faraday.



Capacitance of a capacitor

The other units of capacitance:

Mili-farad (mF): $\xrightarrow{\times 10^{-3}}$ Farad (F)

Micro-farad (μF): $\xrightarrow{\times 10^{-6}}$ Farad (F)

Nano-farad (nF): $\xrightarrow{\times 10^{-9}}$ Farad (F)

Capacitance of a capacitor

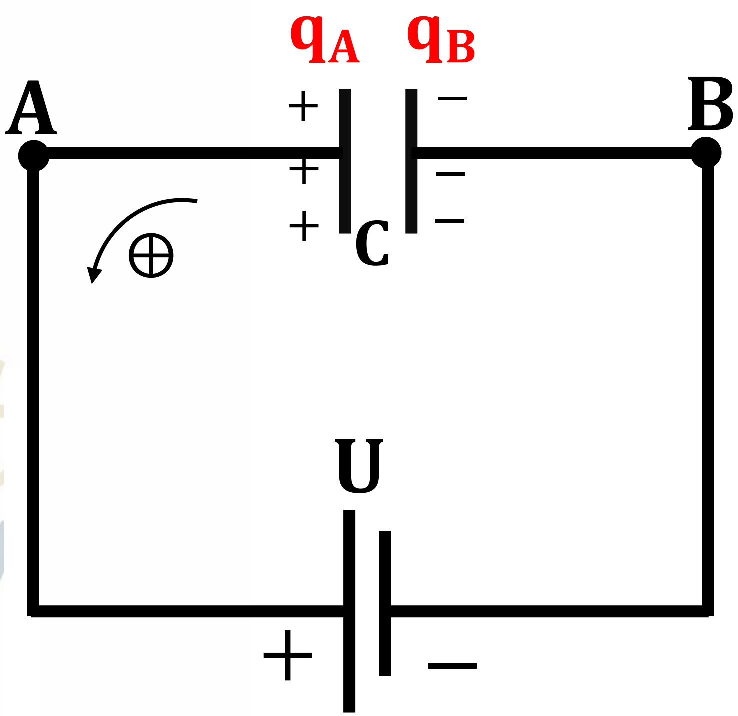
The charge of a capacitor is: $q = q_A = -q_B$

The voltage (U_{AB}) across the capacitor is proportional to q

For plate A : $q_A = CU_{AB}$

For plate B : $q_B = CU_{BA}$

$U_{AB} = -U_{BA}$ and $q_A = -q_B$

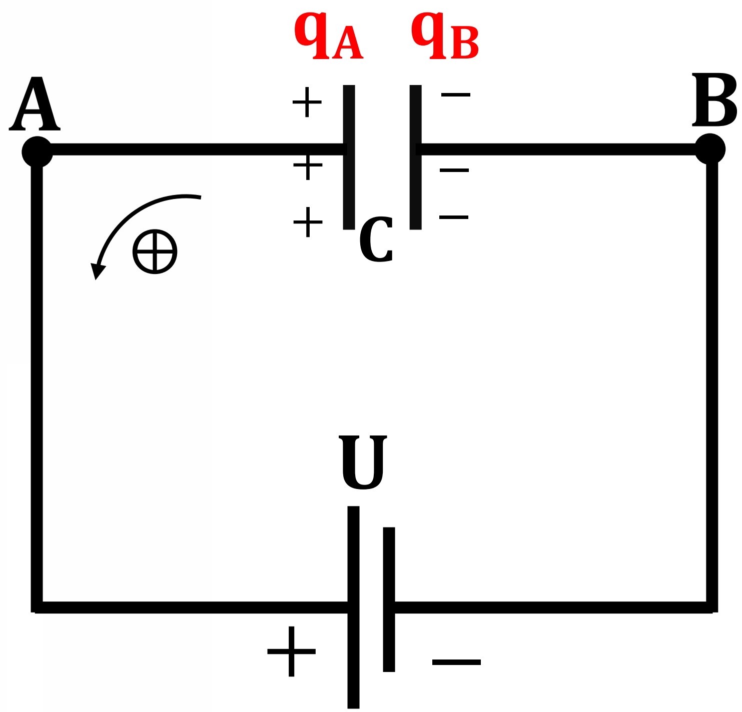


Capacitance of a capacitor

In general, the capacitance of a capacitor is given by:

$$q = C.u_C$$

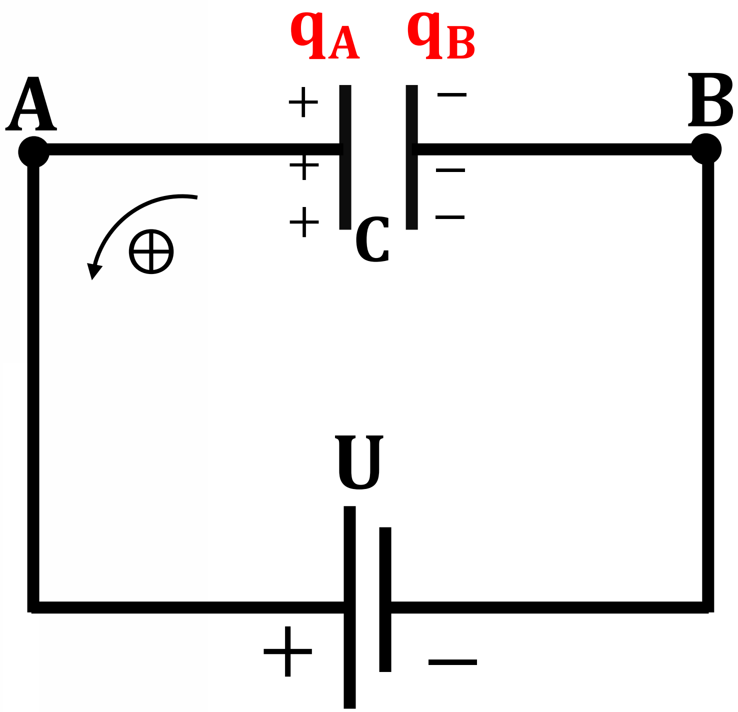
- **U:** Voltage, expressed in volts "V"
- **C:** Capacitance of a capacitor, expressed in farads "F"
- **q :** amount of charge, expressed in coulombs "C"



Energy stored in a capacitor

The energy stored in a capacitor, of capacitance C, and of charge q under a voltage u is given by:

$$w = \frac{1}{2} C {u_C}^2$$



C: The capacitance of the capacitor, in (F)

u_C: voltage across the capacitor, in (V)

W: energy stored in the capacitor, in (J)

Energy stored in a capacitor

$$W = \frac{1}{2} C {u_C}^2 \;\dots\dots (1)$$

But $q = c u_C$ ⟹ $u_C = \frac{q}{C}$

Substitute in (1)

$$W = \frac{1}{2} C \left[\frac{q}{C}\right]^2 \Rightarrow W = \frac{1}{2} C \frac{q^2}{C^2} \Rightarrow W = \frac{1}{2} \cdot \frac{q^2}{C}$$

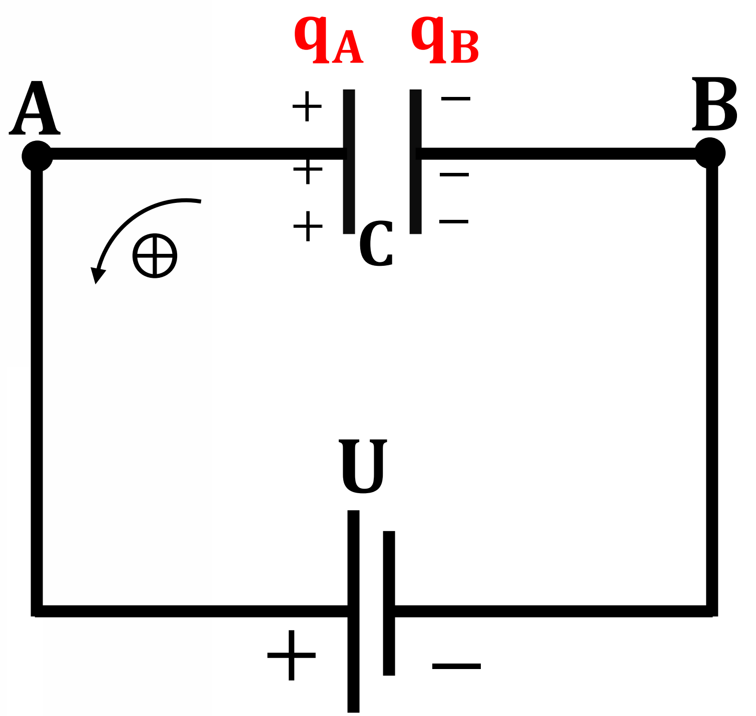
- ***q*:** amount of charge stored in the capacitor in (C)
- **C:** The capacitance of the capacitor, in (F)
- **W:** energy stored in the capacitor, in (J)

Energy stored in a capacitor

Application 1:

A capacitor with a capacity of 5000 µF is charged under a voltage of 12 V.

1) Calculate the accumulated charge in the capacitor.
2) Calculate the energy stored in this capacitor.



Energy stored in a capacitor

$C = 5000\ \mu F; u = 12V$

1)Calculate the accumulated charge in the capacitor.

The accumulated charge stored in the capacitor is:

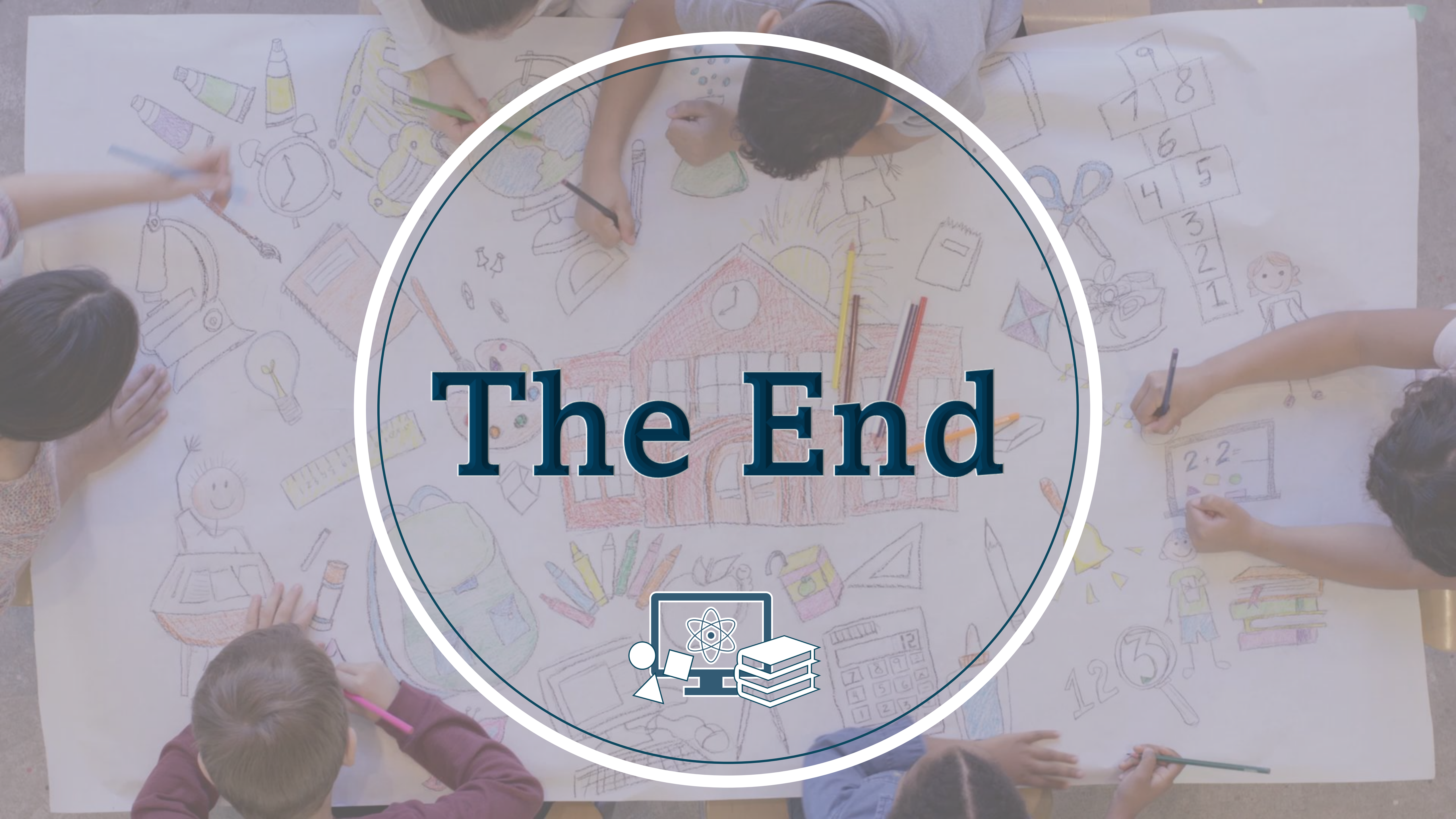
$q = C \times u = 5000 \times 10^{-6} \times 12 \Rightarrow q = 6 \times 10^{-2} C$

2)Calculate the energy stored in this capacitor.

The energy stored in the capacitor is: $w = \frac{1}{2} Cu^2$

$$W = 0.5 \times (5000 \times 10^{-6}) \times (12)^2$$

$$W = 36 \times 10^{-2} J$$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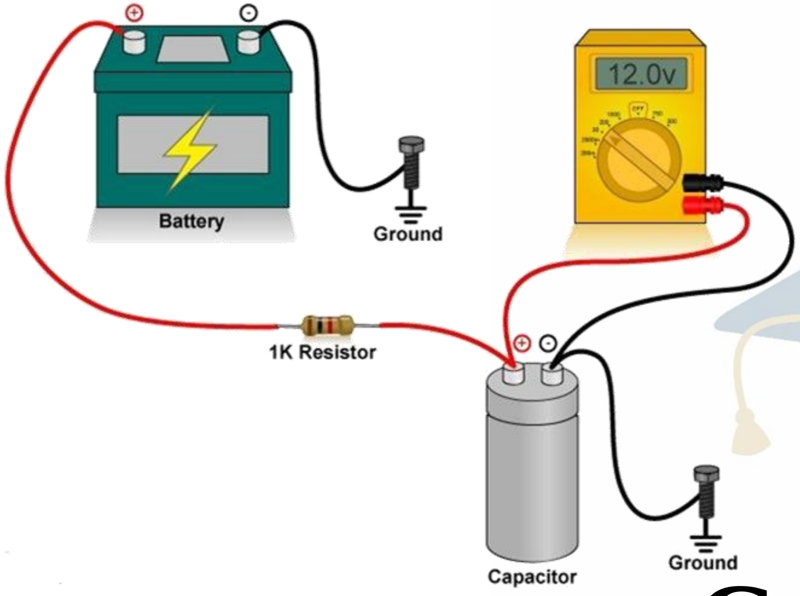


Grade 12 LS – Physics

Chapter 10 -A

Capacitor with a L.F.G of square signal

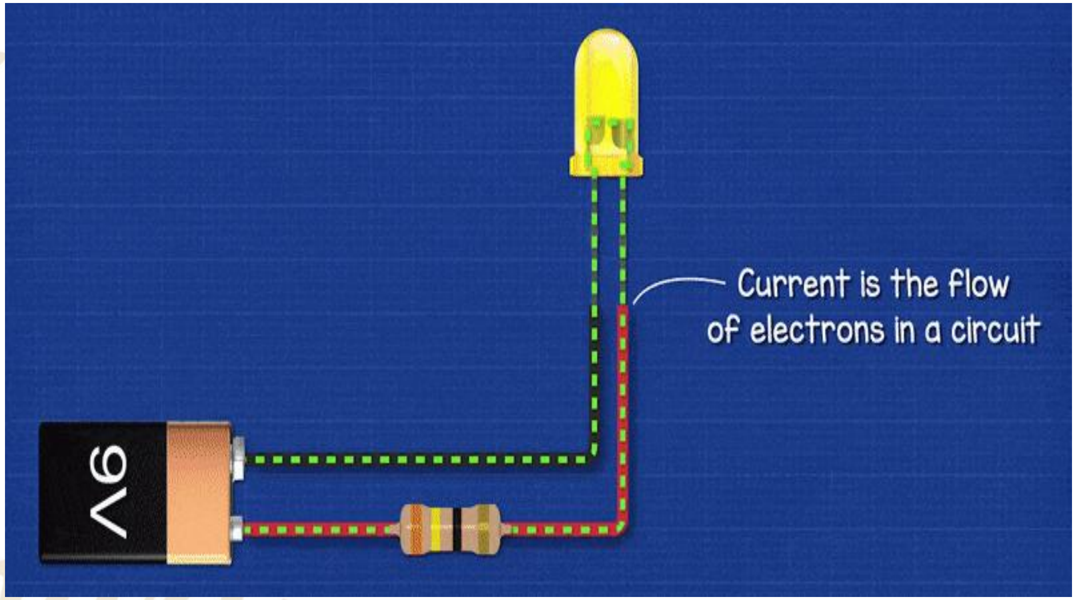
Prepared & Presented by: Mr. Mohamad Seif

OBJECTIVES

1. Relation between q and i
2. Using the oscilloscope to visualize the voltage
3. The low frequency generator (LFG)

Relation between q and *i*

The electric current is the flow of charges (electrons) in a certain area per unit time.



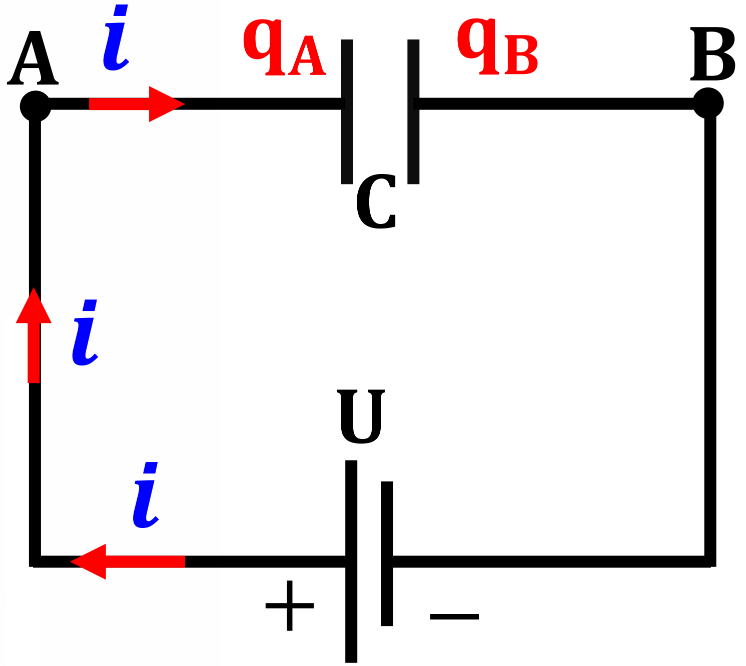
The instantaneous expression of the current could be written:

$$i = \left| \frac{dq}{dt} \right|$$

Relation between q and i

If the capacitor collects charges, the charge q increases:

$$\frac{dq}{dt} > 0 \Rightarrow i = +\frac{dq}{dt}$$



But $q = Cu_C$ $\Rightarrow$ $i = +\frac{dCu_C}{dt}$

$$i = +C\frac{du_C}{dt}$$

Relation between q and i

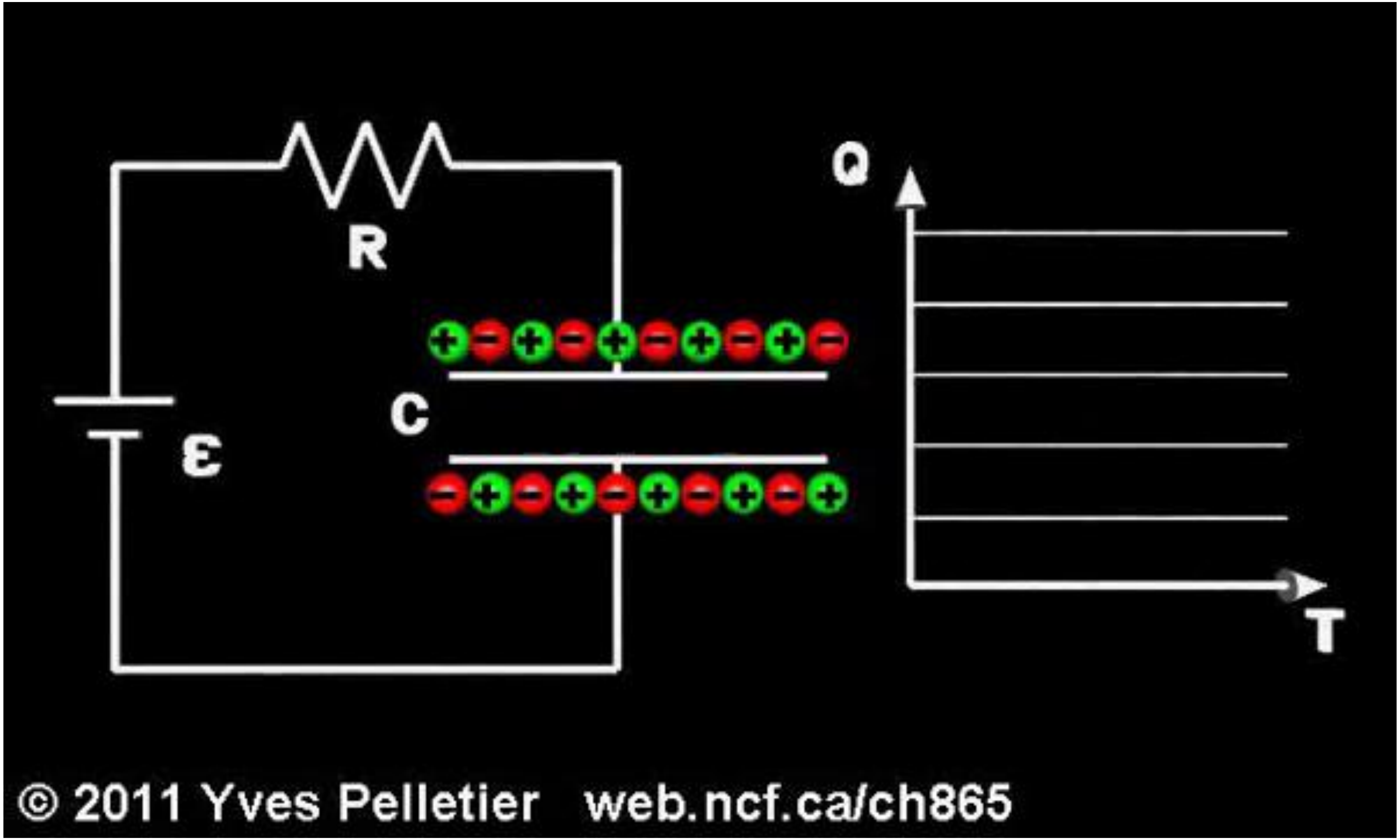
If the capacitor loses charges, the charge q decreases:

$$\frac{dq}{dt} < 0 \quad \Rightarrow \quad i = -\frac{dq}{dt}$$

But $q = Cu_C \quad \Rightarrow \quad i = -\frac{dCu_C}{dt}$

$$i = -C\frac{du_C}{dt}$$

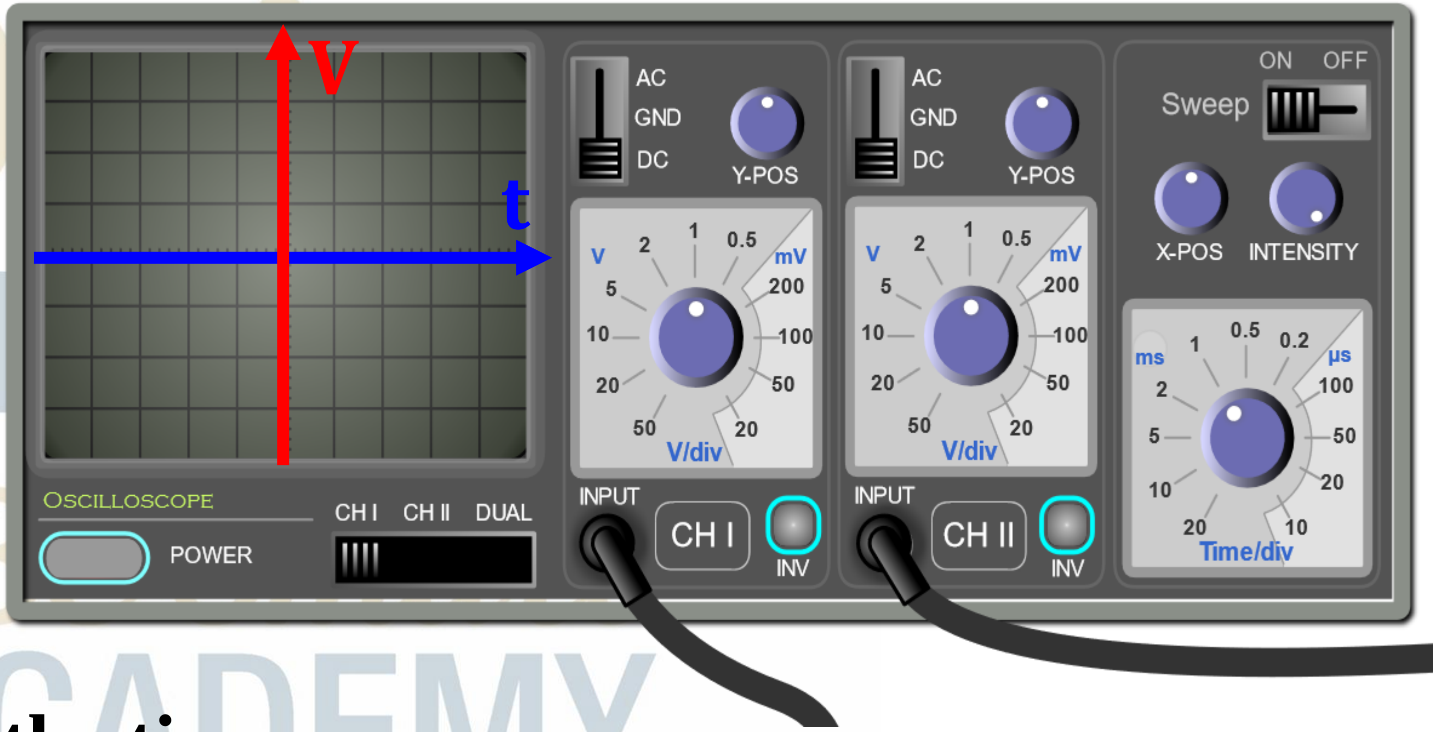
Flow of charges in the capacitor



Using the oscilloscope to visualize the voltage

The Oscilloscope is used to visualize and measure the voltage of electric component connected in parallel.

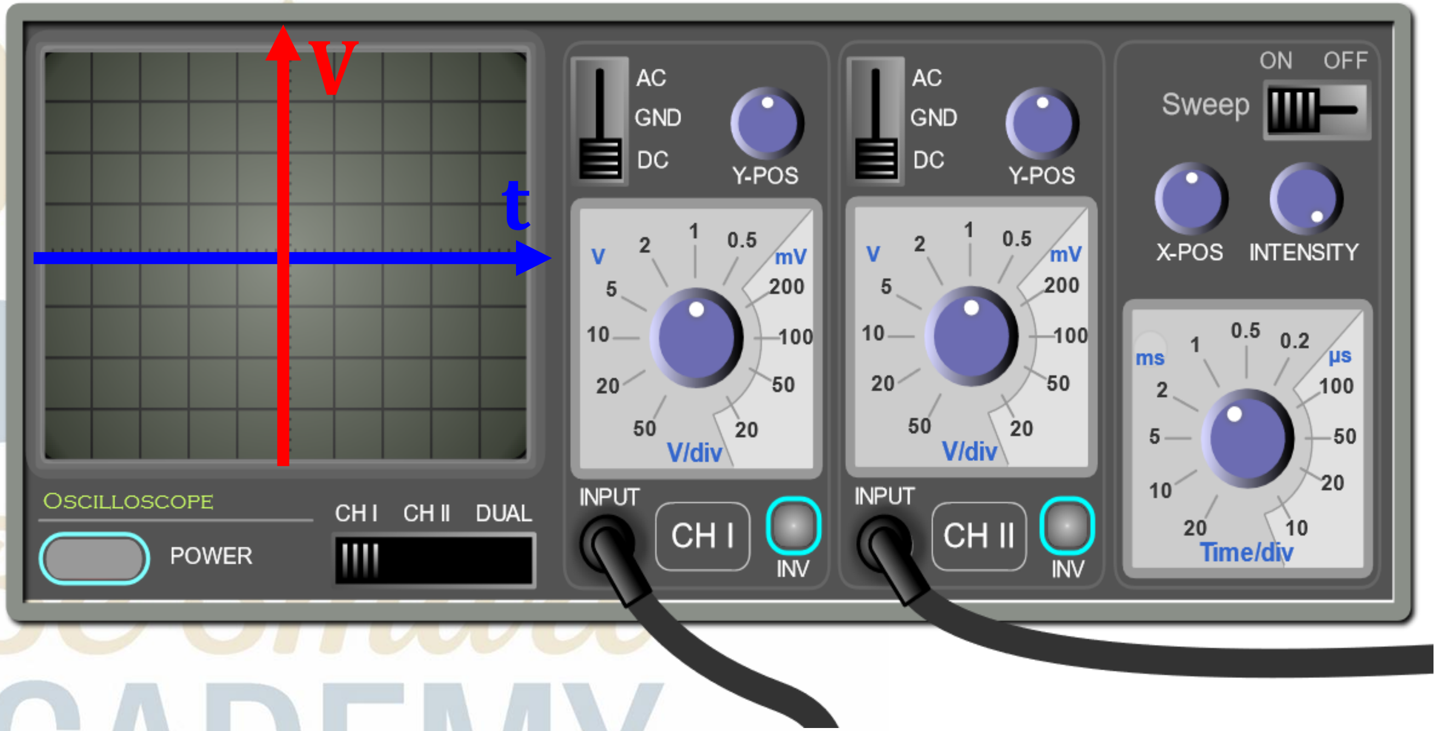
The displayed graph on the screen of the oscilloscope is called oscillogram.



- **The horizontal axis represents the time.**
- **The vertical axis (y) represents the voltage in volts.**

Using the oscilloscope to visualize the voltage

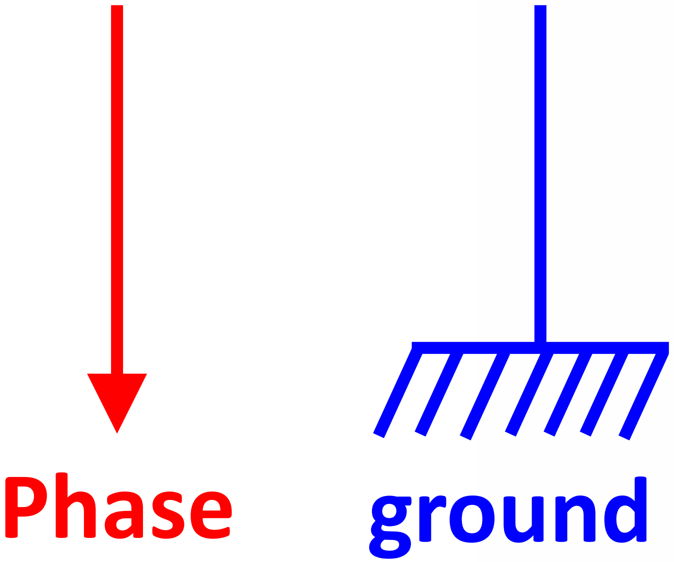
The oscilloscope has two terminals: phase (channel) and ground (mass).

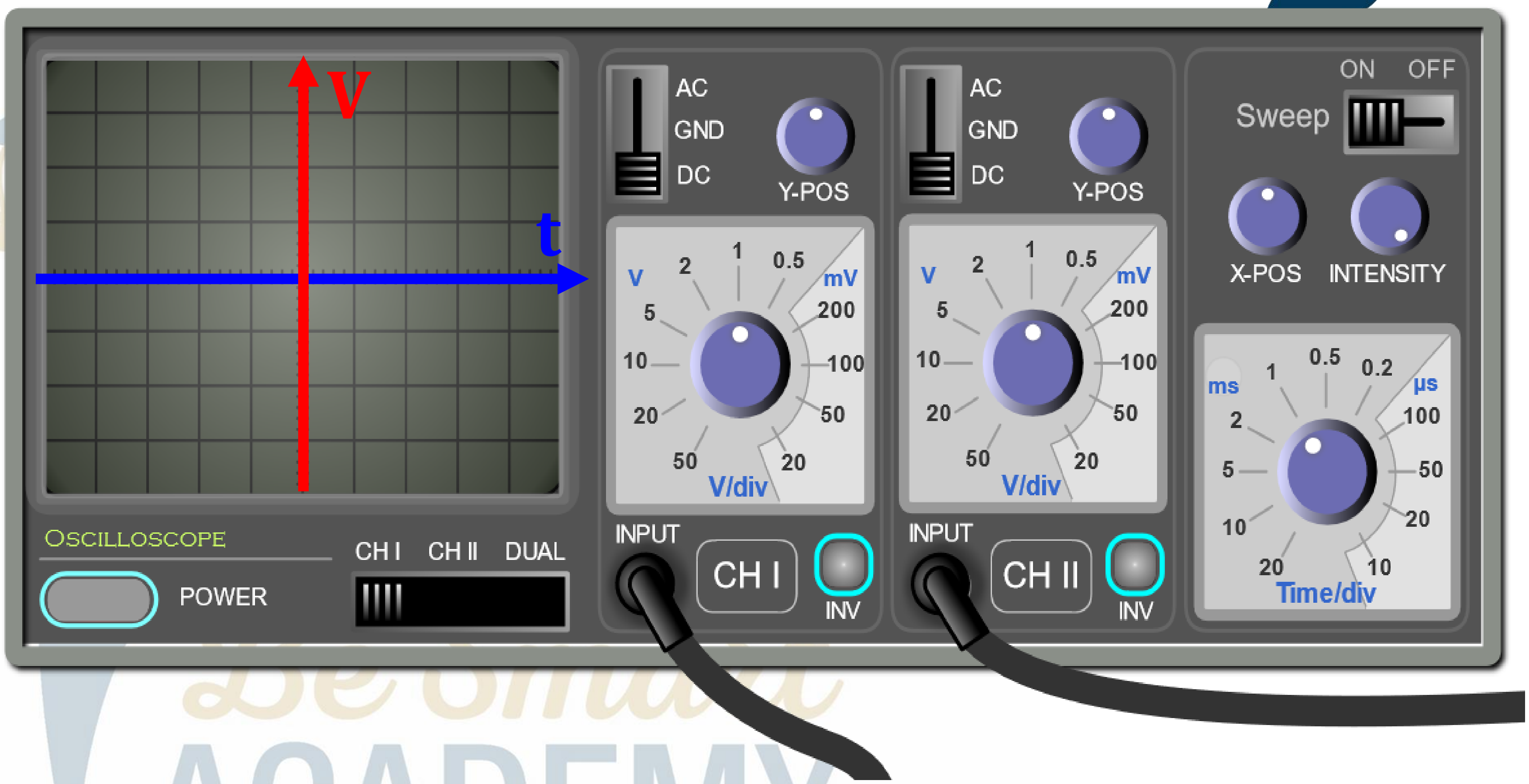


Vertical sensitivity S_V (V/div) which regulates the scale of the voltage.

Horizontal sensitivity (S_h): expressed in (ms/div). it gives the number of milliseconds represented by one division

Using the oscilloscope to visualize the voltage





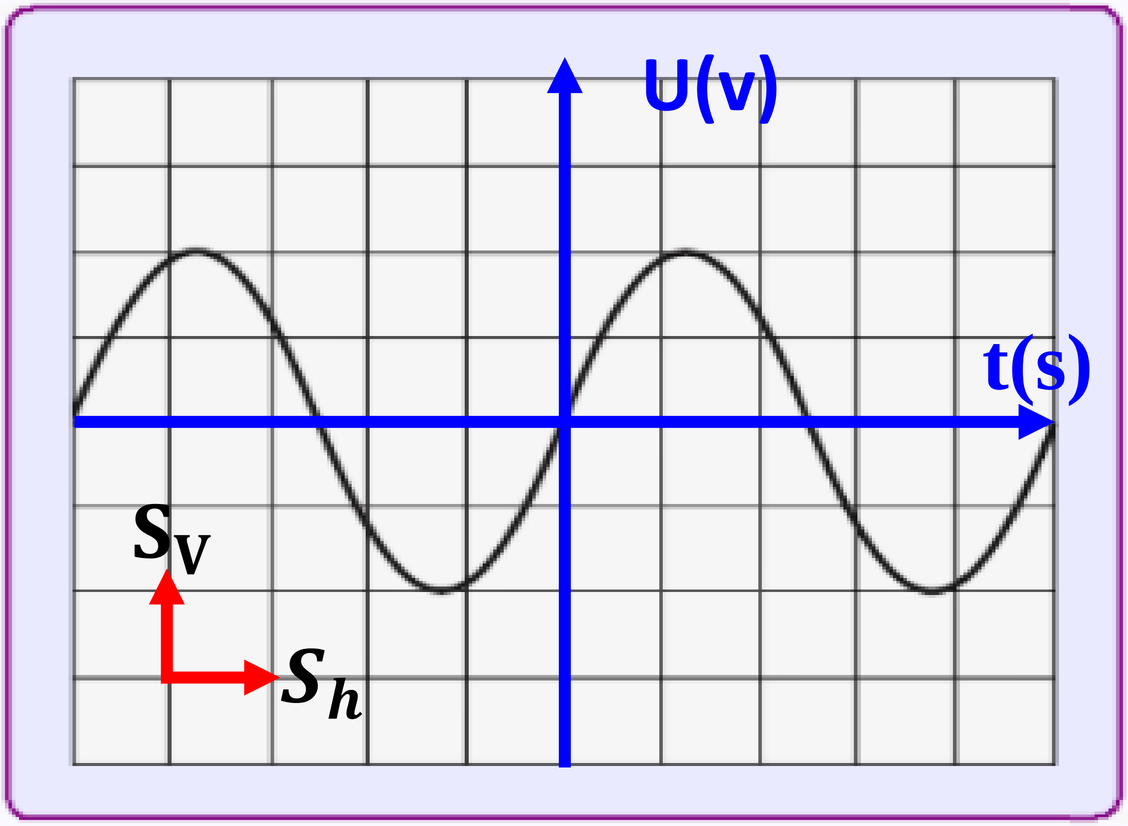
The oscilloscope reads from phase to ground.

Using the oscilloscope to visualize the voltage

The value of the voltage is given by:

$$u = S_V \times y$$

- y: number of divisions on the y axis
- S_V: vertical sensitivity (V / div).
- u: voltage across the dipole (V).

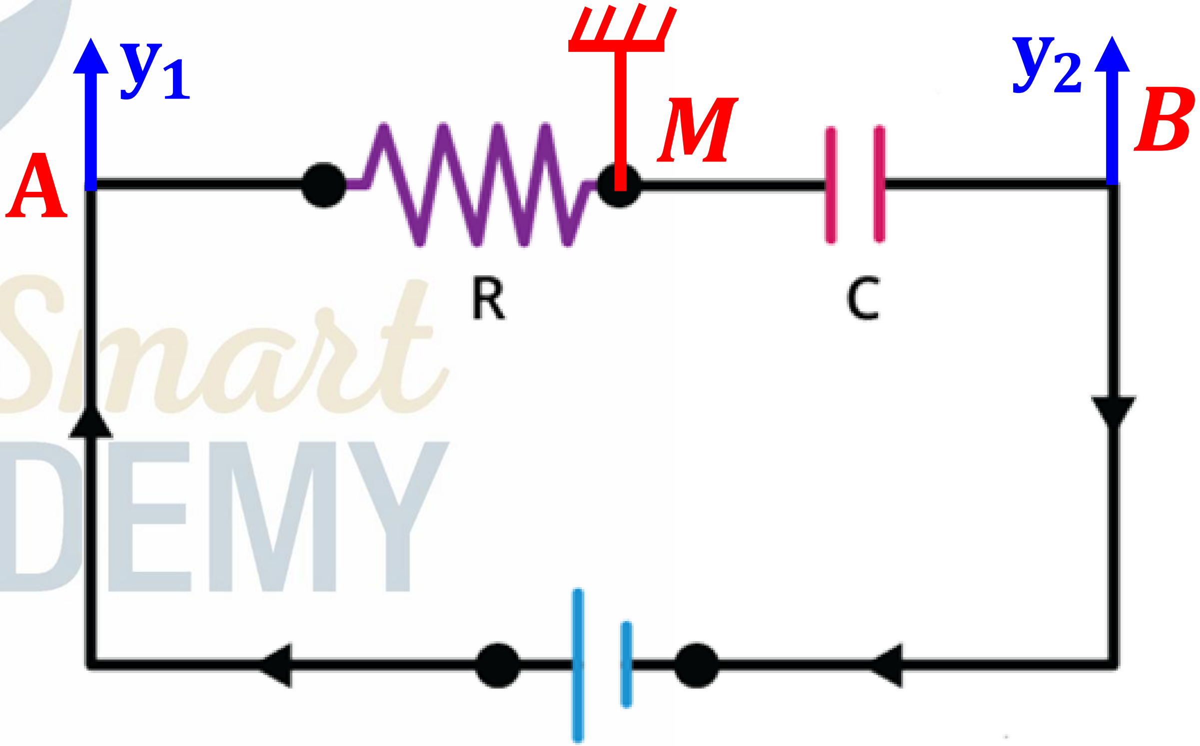


Using the oscilloscope to visualize the voltage

Application 2:

An oscilloscope allows us to display the voltage $u_{AM} = u_R$ across the resistor and the voltage $u_{BM} = u_C$ across the capacitor.

1) Show the connections of the oscilloscope on the figure.



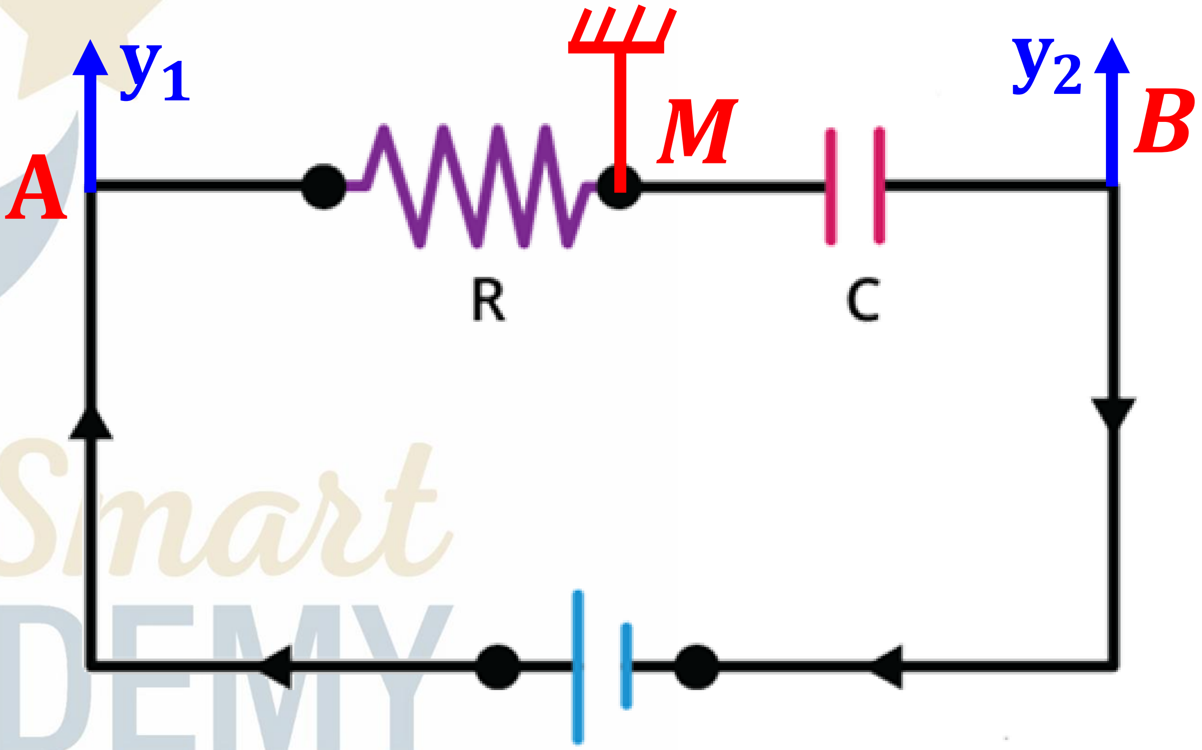
u_{AM} on channel y_1

u_{BM} on channel y_2.

Using the oscilloscope to visualize the voltage

2)Which button must be activated?

To measure u_{MB} you should press the <invert> button on channel y_2 of the oscilloscope.



The low frequency generator (LFG)

Law frequency generator (LFG):

LFG is a signal generator device that is used to produce signals of varying amplitude and frequency.

The adjustable frequency range of the generated signal falls between 100Hz to 1MHz

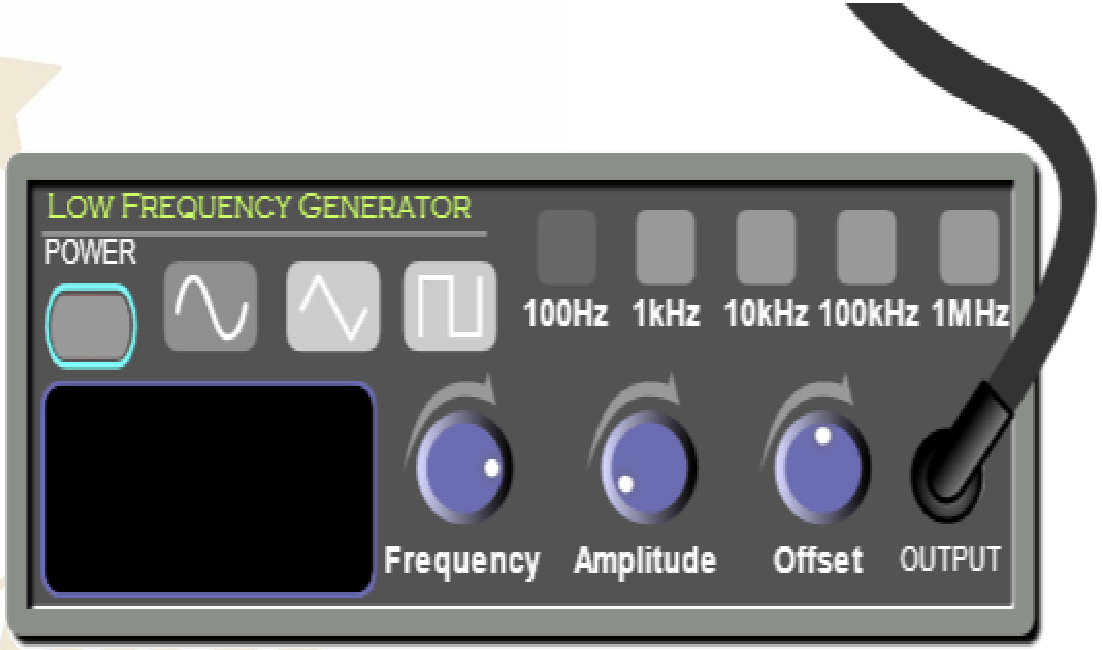
The amplitude can be adjusted from some millivolts to volts.

The low frequency generator (LFG)

It is usually a source for generating sinusoidal signals.

However, it can also produce a signal in three different forms:

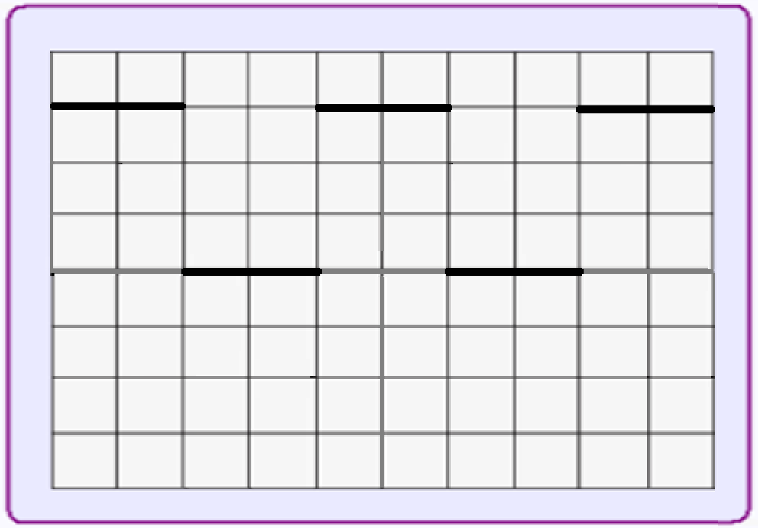
- Sinusoidal wave
- Triangular wave
- Rectangular wave



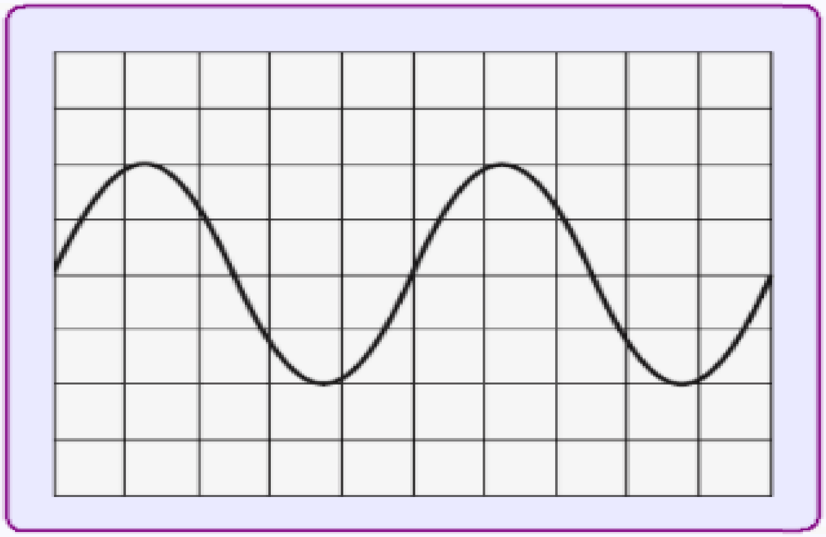
The low frequency generator (LFG)

Types of generated signals

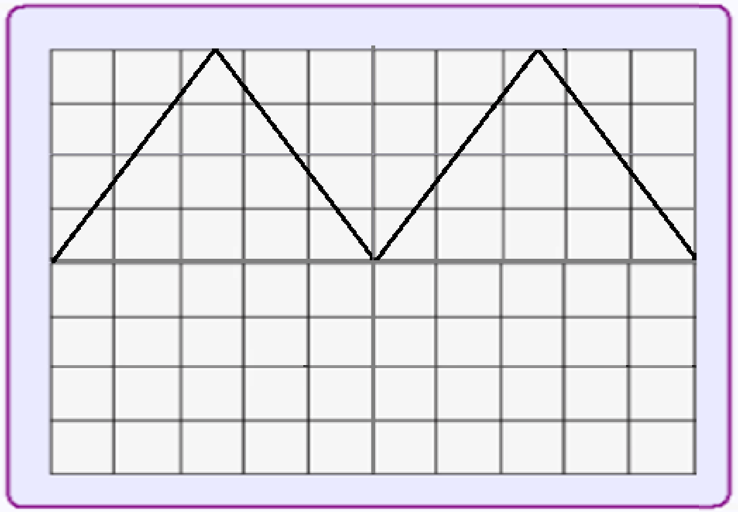
Square wave



Sinusoidal wave



Triangular wave



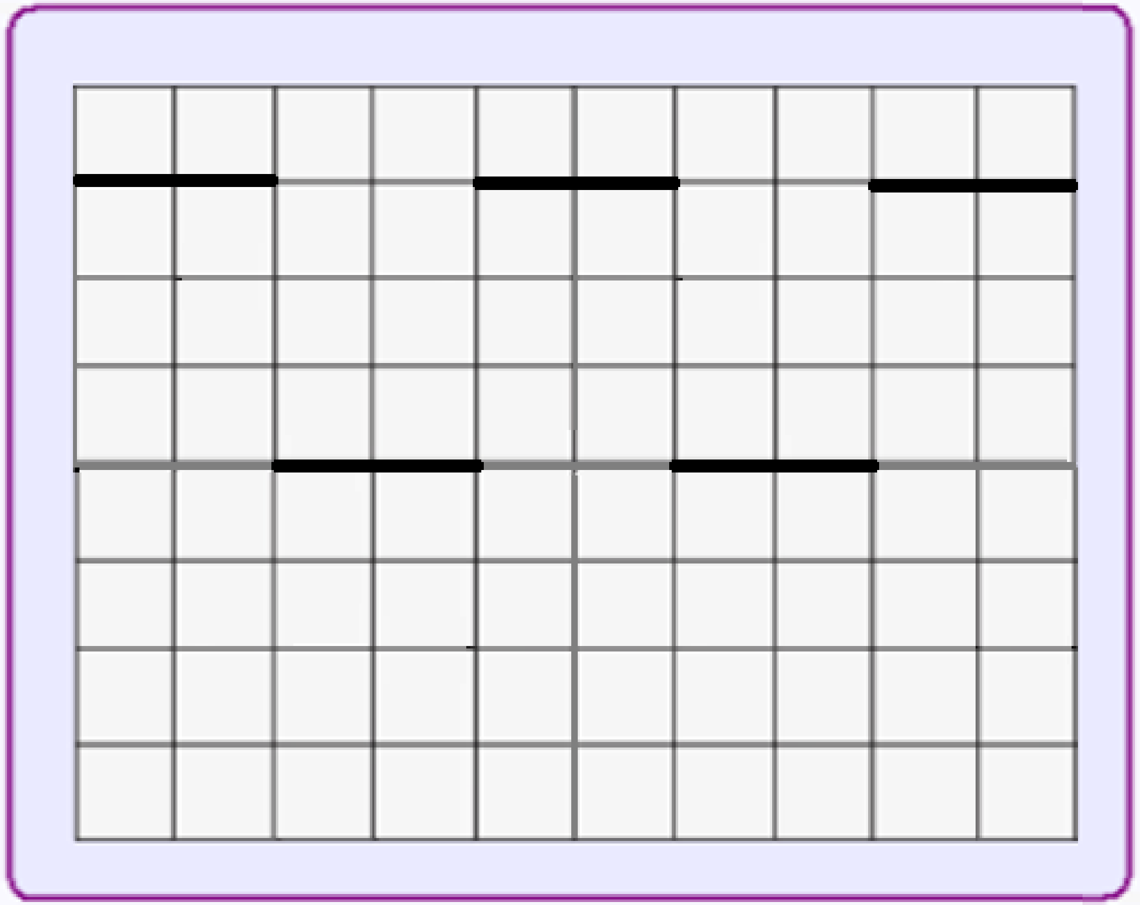
The low frequency generator (LFG)

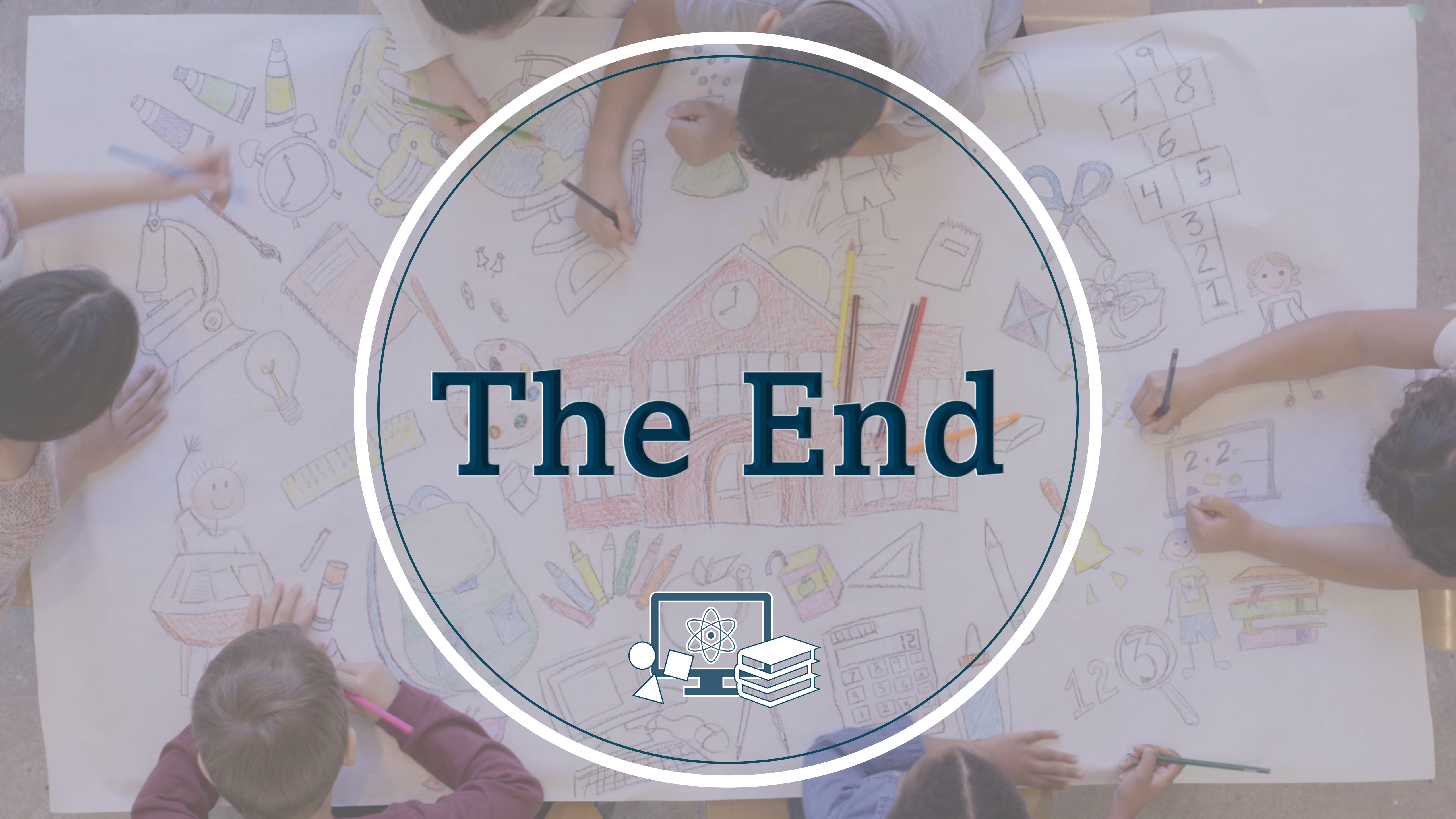
In this lesson we will focus on square signals of LFG.

The value of the square alternating voltage varies periodically over time.

The expression of this voltage is of the form:

$$u_G = \begin{cases} E & 0 \leq t \leq \frac{T}{2} \\ 0 & \frac{T}{2} \leq t \leq T \end{cases}$$

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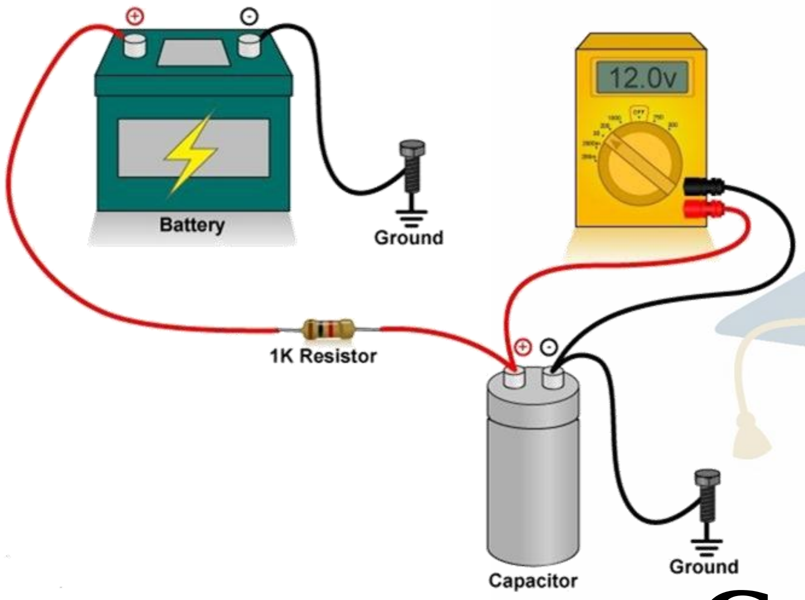


Grade 12 LS – Physics

Chapter 10 -A

Capacitor with a L.F.G of square signal

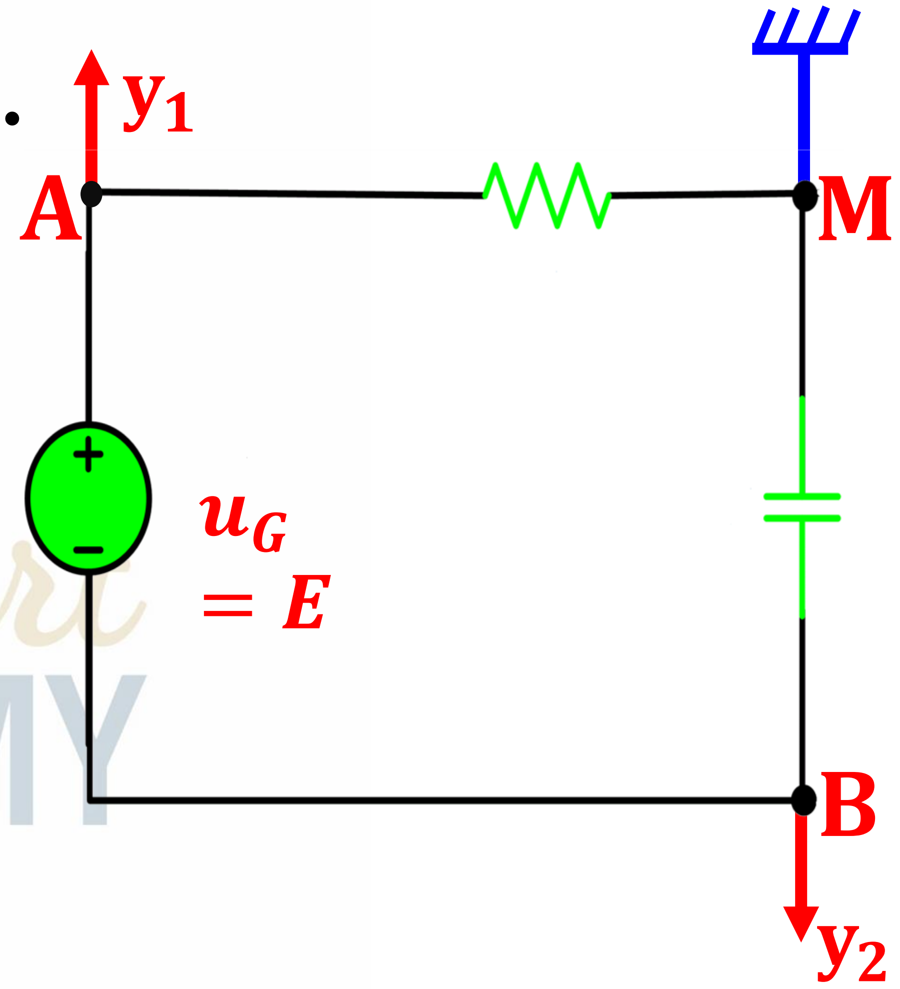
Prepared & Presented by: Mr. Mohamad Seif

OBJECTIVES

1. Study the charging of a capacitor experimentally
2. Study the differential equation in u_C (theoretically)

Study of charging of a capacitor experimentally

- Channel y_1 displays the voltage $u_{AM} = u_R$.
- Channel y_2 displays the voltage $u_{BM} = u_C$.
- The "inv" button should be actived on channel 2 to display u_{MB}.

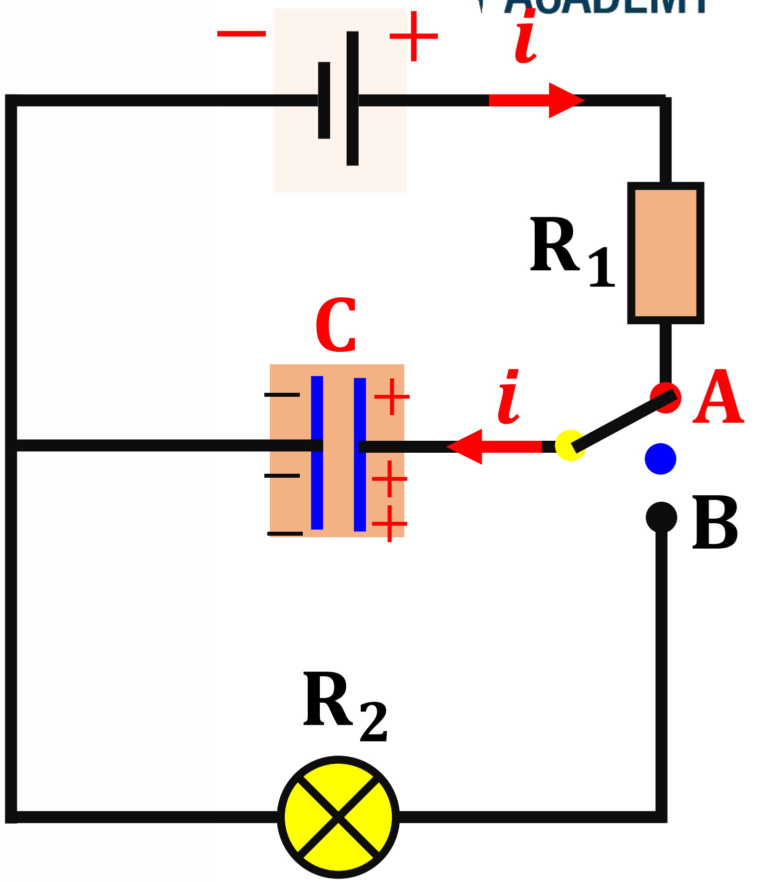


Study of charging of a capacitor experimentally

The positive pole of the generator attracts the free electrons of the armature A.

Armature A loses these electrons and becomes positively charged; $q_A > 0$.

Electrons move from the negative pole of the generator to the armature B.



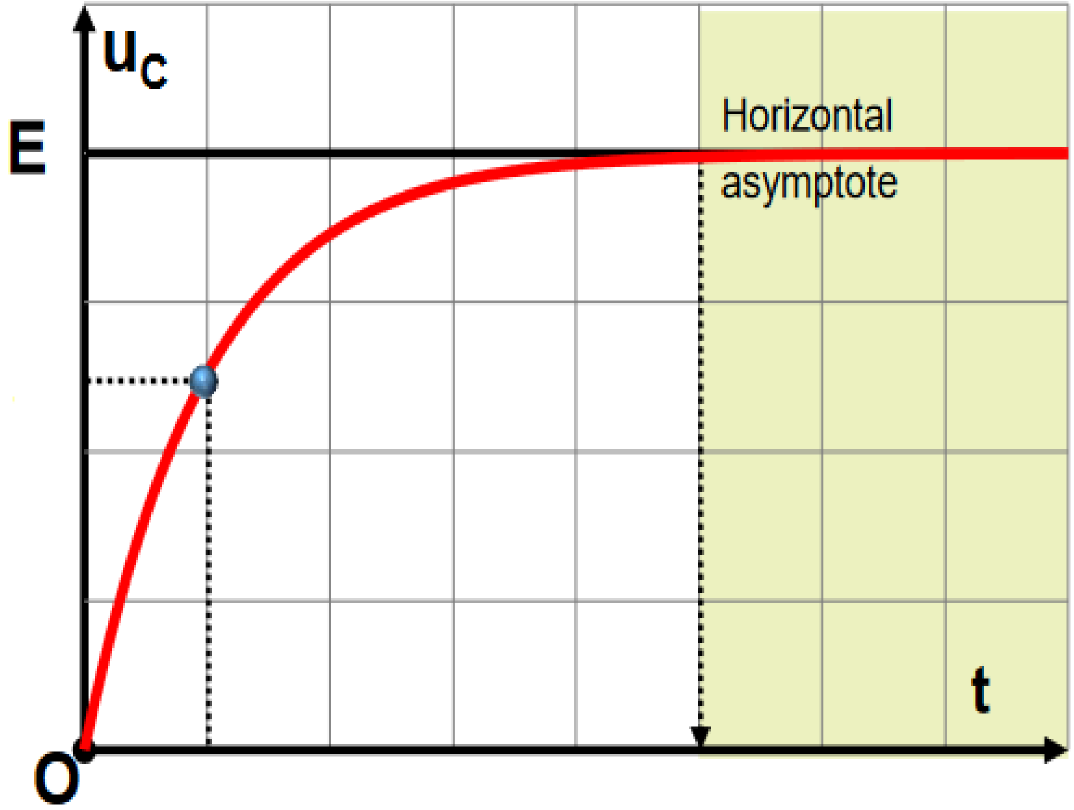
Armature B gains these electrons and becomes negatively charged whose quantity of charge $q_B < 0$ such that: $q_A = -q_B$.

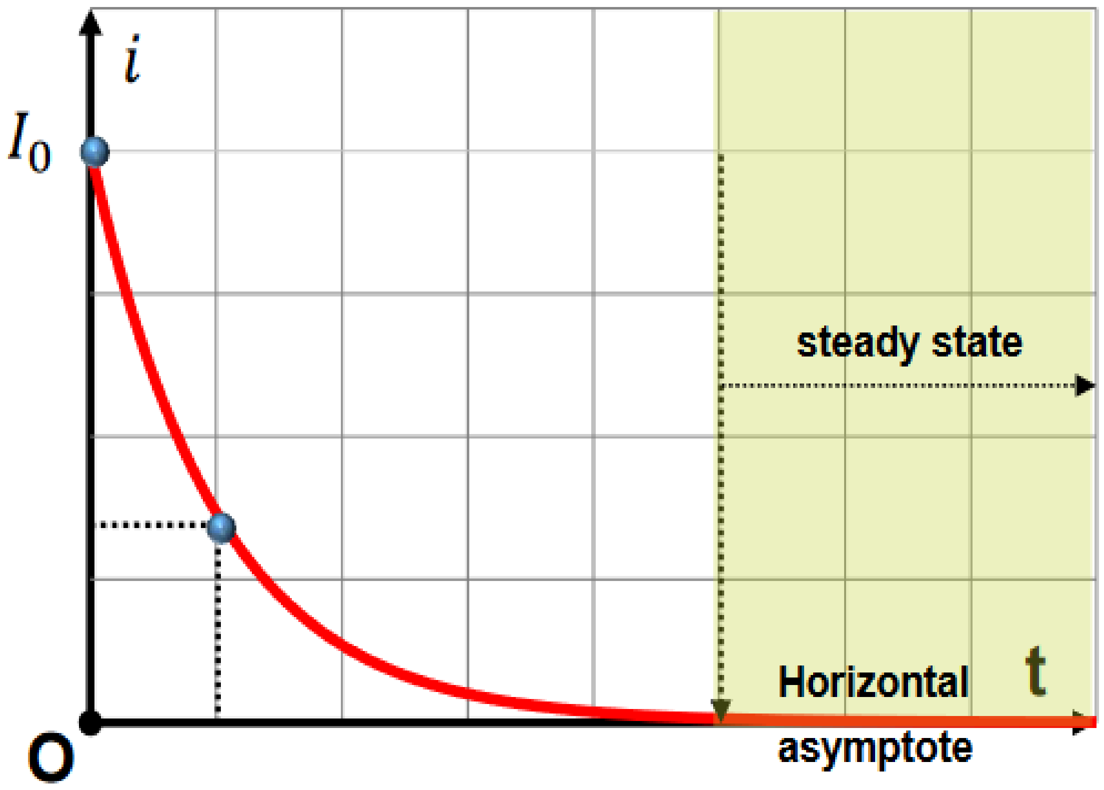
Let's first understand the Charging process

Charging a capacitor: Experimental study

For the switch at point (A): $u_G = E$ $\quad 0 \leq t \leq \frac{T}{2}$:

The following curves are observed on the screen of the oscilloscope.





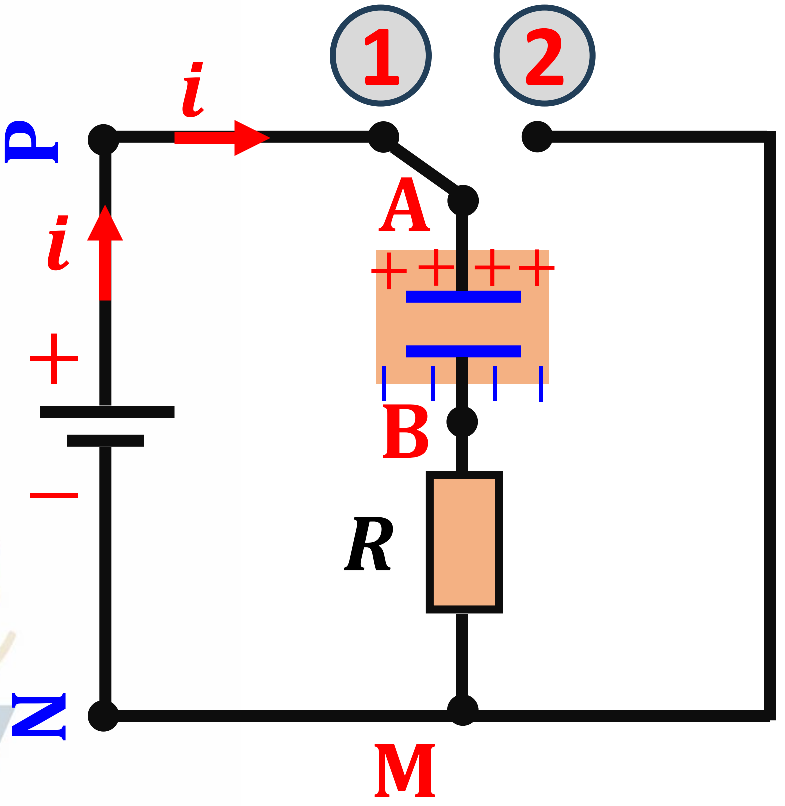
Study of charging of a capacitor theoretically

The electric circuit consists of:

- An ideal generator of voltage $u_G = E$
- A resistor of resistance R.
- A neutral capacitor of capacitance C
- A switch K

At an instant $t = 0$, the switch K turns to position (1):

The charging process of the capacitor starts



Study of charging of a capacitor theoretically

Determine the first order differential equation that governs the variation of u_C.

According to the law of addition of voltages:

$$u_G = u_C + u_R$$

where: $u_G = E$ and $u_R = Ri$

but $i = +\frac{dq}{dt}$ And $q = Cu_C$

$$i = \frac{d(Cu_C)}{dt} \Rightarrow i = C\frac{du_C}{dt}$$

$$E = u_C + Ri$$

$$E = u_C + RC\frac{du_C}{dt}$$

Study of charging of a capacitor theoretically

The solution of the differential equation is:

$$u_C = E\left(1 - e^{-\frac{t}{\tau}}\right)$$

At t=0

$$u_C = E\left(1 - e^{-\frac{0}{\tau}}\right)$$

$$u_C = E(1 - 1)$$

$$u_C = 0$$

Study of charging of a capacitor theoretically

$$u_C = E\left(1 - e^{-\frac{t}{\tau}}\right)$$

At $t = \tau$

$$u_C = E\left(1 - e^{-\frac{\tau}{\tau}}\right)$$

$$u_C = E\left(1 - e^{-1}\right) \Rightarrow u_C = E(1 - 0.37)$$

$$u_C = 0.63E \qquad u_C = 63\%E$$

$t = \tau$: is the time needed to charge the capacitor 63% of its maximum value (E)

Study of charging of a capacitor theoretically

$$u_C = E\left(1 - e^{-\frac{t}{\tau}}\right)$$

At $t = 5\tau$

$$u_C = E\left[1 - e^{-\frac{5\tau}{\tau}}\right]$$

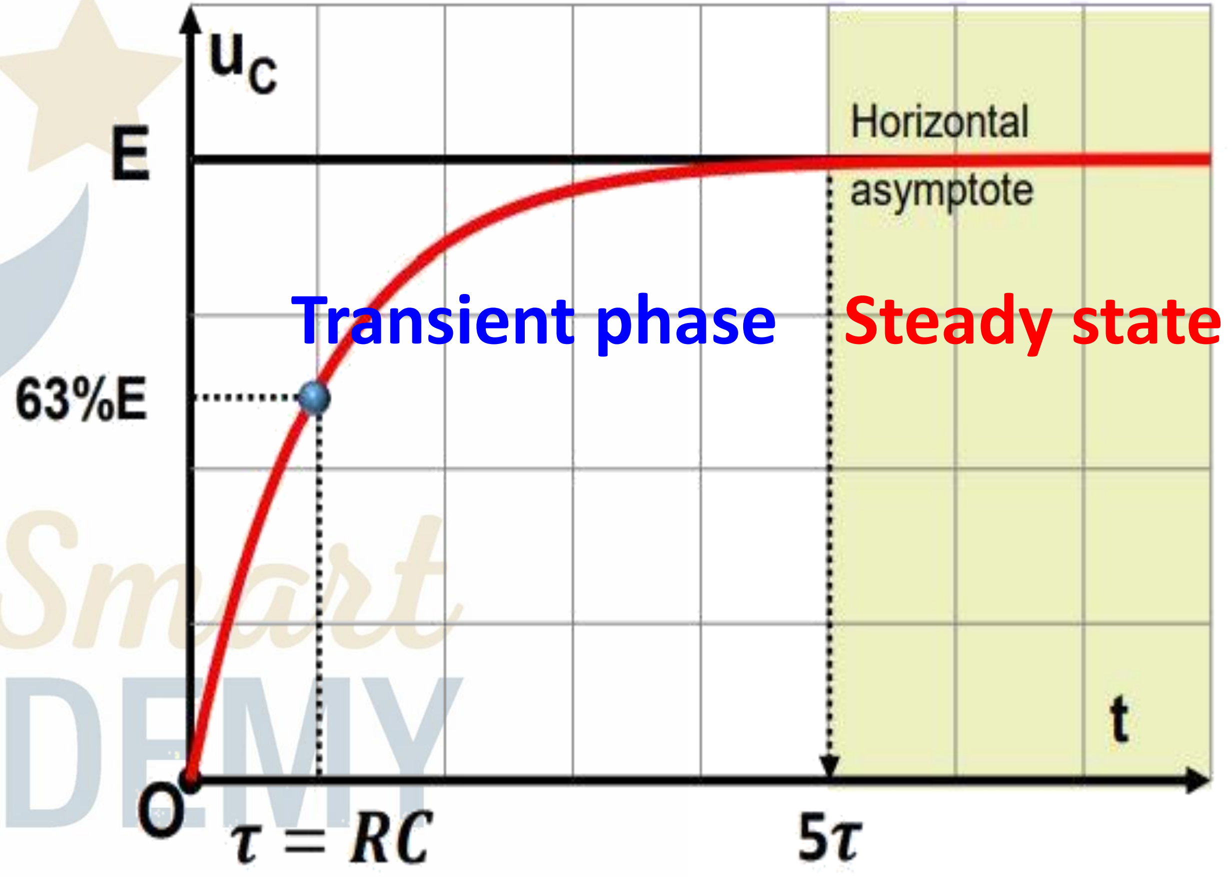
$$u_C = E\left(1 - e^{-5}\right) \Rightarrow u_C = E(1 - 0.0067)$$

$$u_C = 0.99E \approx E$$

At $t = 5\tau$, the capacitor is practically completely charged

Study of charging of a capacitor theoretically

Summary		
$t = 0$	$t = \tau$	$t = 5\tau$
$u_C = 0$	$u_C = 0.63E$	$u_C = E$



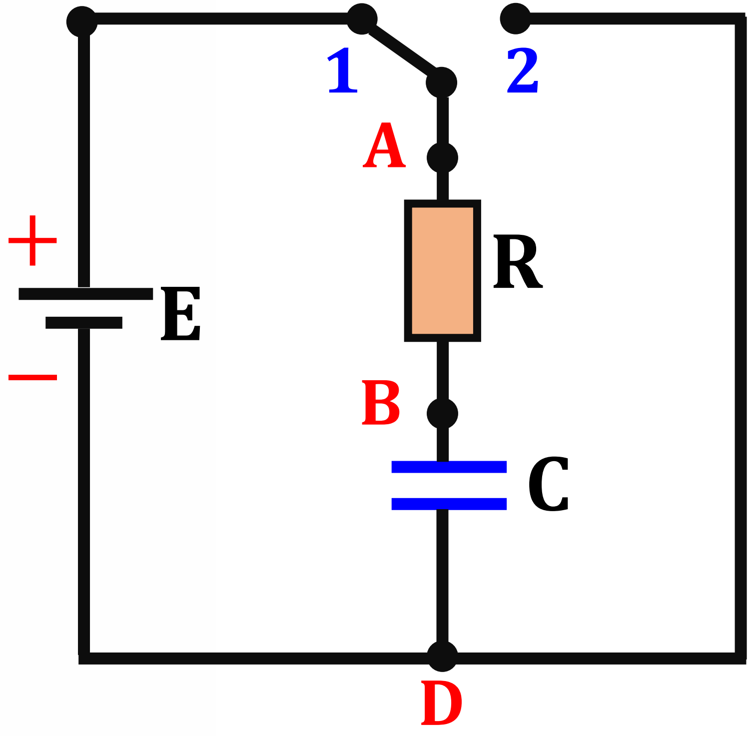
Study of charging of a capacitor theoretically

Application 3:

Consider the electric circuit that consists of an ideal generator delivering, between its terminals, a constant voltage of value E, a capacitor of capacitance C, a resistor of resistance R and of a switch K.

The capacitor is initially neutral.

At time $t_0 = 0$, we place K at position (1); the capacitor charging phenomenon begins.



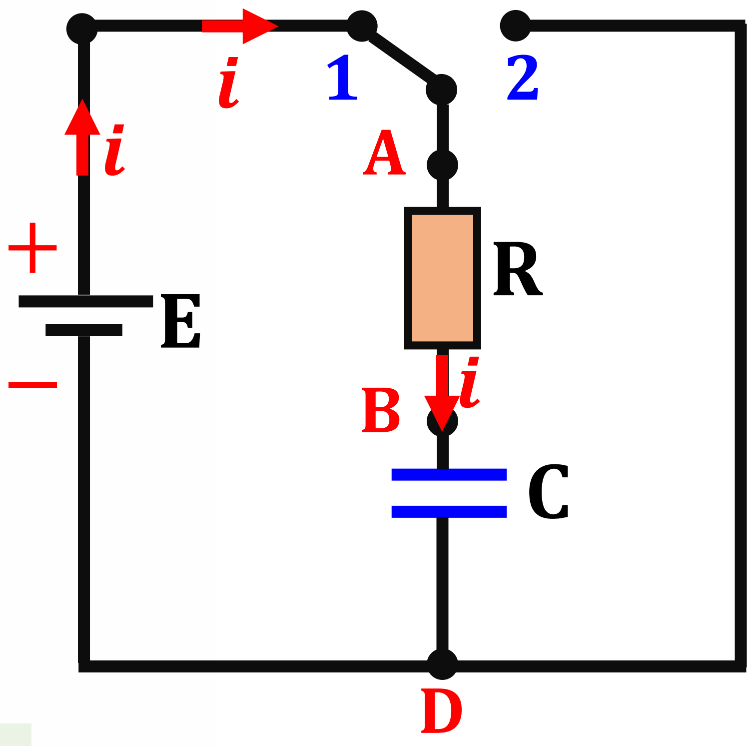
Study of charging of a capacitor theoretically

1) Represent the direction of the current on the circuit.
2) Determine the relationship between i, C and u_C.
3) Establish the differential equation that describes the variation of the voltage u_C, with respect to time.
4) Show that $u_C = E(1 - e^{-\frac{t}{\tau}})$ is a solution of the differential equation in u_C .
5) The solution of the above differential equation is of the form: $u_C = A + Be^{-\frac{t}{\tau}}$. Determine A and B as a function of E, and τ as a function of RC.

Study of charging of a capacitor theoretically

1) Represent the direction of the current on the circuit.

The direction of the current leaves the positive pole and enters the negative pole of the generator.



2) Determine the relationship between i, C and u_C.

$$i = +\frac{dq}{dt}$$

and $q = C.u_C$

$$i = \frac{d(Cu_C)}{dt}$$

$$i = C\frac{d(u_C)}{dt}$$

Study of charging of a capacitor theoretically

3) Determine the first order differential equation that governs the variation of u_C.

According to the law of addition of voltages:

$$u_G = u_C + u_R$$

where: $u_G = \mathrm{E}$ and $u_R = Ri$

but $i = +\frac{dq}{dt}$ And $\mathrm{q} = Cu_C$

$$\mathrm{i} = \frac{d(Cu_C)}{dt} \Rightarrow i = C\frac{du_C}{dt}$$

$$\mathrm{E} = u_C + \mathrm{Ri}$$

$$E = u_C + RC\frac{du_C}{dt}$$

Study of charging of a capacitor theoretically

4)Show that $u_C = E(1 - e^{-\frac{t}{\tau}})$ is a solution of the differential equation in u_C

$$u_C = E - Ee^{-\frac{t}{RC}}$$

$$\frac{du_C}{dt} = +\frac{E}{RC}.e^{-\frac{t}{RC}}$$

Substitute u_C and $\frac{du_C}{dt}$ in differential equation

$$E = u_C + RC\frac{du_C}{dt}$$

$$E = E - E.e^{-\frac{t}{RC}} + RC.\frac{E}{RC}.e^{-\frac{t}{RC}}$$

$$E = E - E.e^{-\frac{t}{RC}} + E.e^{-\frac{t}{RC}}$$

$$E = E$$

$u_C = E(1 - e^{-\frac{t}{\tau}})$ is a solution of the differential equation

Study of charging of a capacitor theoretically

5)The solution of the above differential equation is of the form: $u_C = A + Be^{-\frac{t}{\tau}}$. Determine A and B in terms of E, and τ as a function of RC.

$$u_C = A + Be^{-\frac{t}{\tau}}$$

$$\frac{du_C}{dt} = -\frac{B}{\tau}e^{-\frac{t}{\tau}}$$

Substitute u_C and $\frac{du_C}{dt}$ in differential equation

$$E = u_C + RC\frac{du_C}{dt}$$

$$E = A + Be^{-\frac{t}{\tau}} - RC.\frac{B}{\tau}e^{-\frac{t}{\tau}}$$

$$0 = -E + A + B\,e^{-\frac{t}{\tau}} - \frac{RCB}{\tau}e^{-\frac{t}{\tau}}$$

Study of charging of a capacitor theoretically

$$0 = -E + A + \mathrm{B}\, e^{-\frac{t}{\tau}} - \frac{RCB}{\tau} e^{-\frac{t}{\tau}}$$

$$0 = (-E + A) + \mathrm{B}\, e^{-\frac{t}{\tau}} \left[1 - \frac{RC}{\tau}\right]$$

$$-E + A = 0 \Rightarrow A = E$$

$$-\frac{RC}{\tau} + 1 = 0 \Rightarrow 1 = \frac{RC}{\tau}$$

$$\tau = RC$$

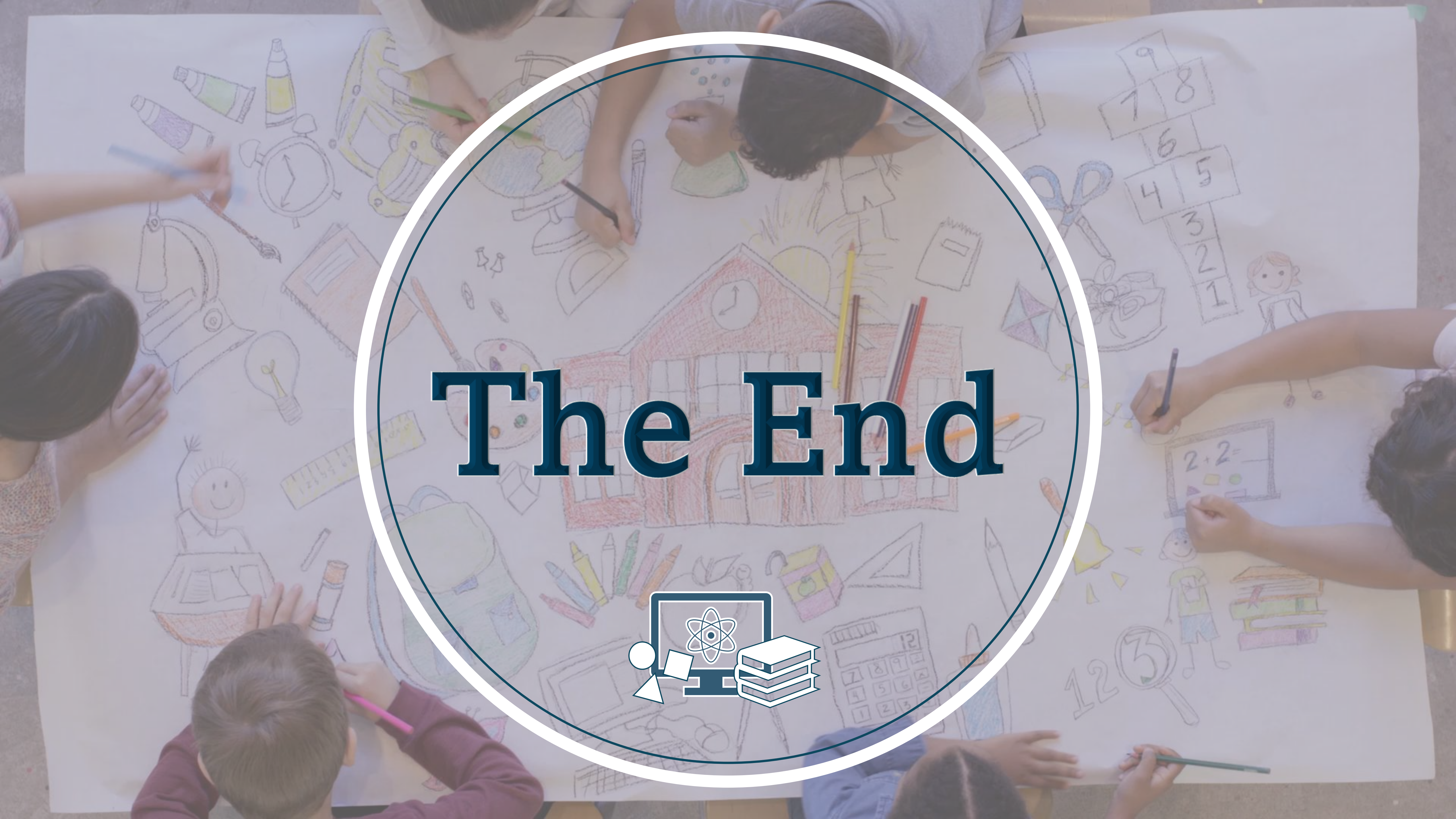
$$u_C = E + B.e^{-\frac{t}{RC}}$$

At $\mathrm{t} = 0;\ u_C = 0$

$$0 = E + B.e^{-\frac{0}{RC}}$$

$$0 = E + B \Rightarrow B = -E$$

$$u_C = E - E.e^{-\frac{t}{RC}}$$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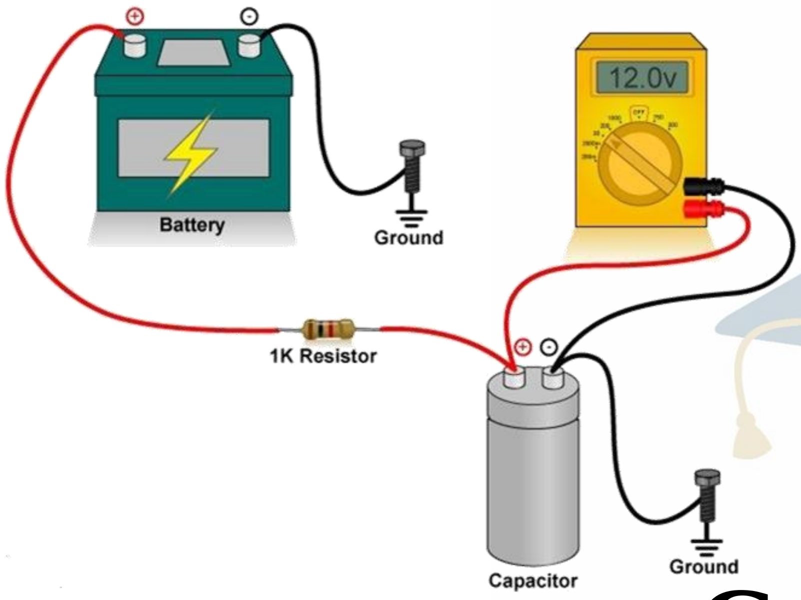


Grade 12 LS – Physics

Chapter 10 -A

Capacitor with a L.F.G of square signal

Prepared & Presented by: Mr. Mohamad Seif

OBJECTIVES

1. Study the differential equation in q in charging phase.
2. Study the solution of the differential equation in q.

Study of charging of a capacitor theoretically

Derive the differential equation that governs the variation of charge q.

Using law of addition of voltages in series:

$$u_G = U_C + U_R$$

$$E = u_C + Ri$$

but $i = +\frac{dq}{dt}$

And $q = Cu_C$ ⇒ $u_C = \frac{q}{C}$

$$E = \frac{q}{C} + R\frac{dq}{dt}$$

$$E.C = q + RC.\frac{dq}{dt}$$

Study of charging of a capacitor theoretically

The solution of the differential equation in terms of q is:

$$q = CE\left(1 - e^{-\frac{t}{\tau}}\right)$$

At $t = 0$

$$q = CE.\left[1 - e^{-\frac{0}{\tau}}\right]$$

$$q = CE(1 - 1)$$

$$q = 0$$

Study of charging of a capacitor theoretically

$$q = CE\left(1 - e^{-\frac{t}{\tau}}\right)$$

At $t = \tau$

$$q = CE.\left[1 - e^{-\frac{\tau}{\tau}}\right]$$

$$q = CE.\left[1 - e^{-1}\right]$$

$$q = CE(1 - 0.37)$$

$$q = 0.63 \times C.E$$

$t = \tau$: is the time needed to charge the capacitor 63% of its maximum charge (q_m).

Study of charging of a capacitor theoretically

$$q = CE\left(1 - e^{-\frac{t}{\tau}}\right)$$

At $t = 5\tau$:

$$q = CE.\left[1 - e^{-\frac{5\tau}{\tau}}\right]$$

$$q = CE.\left[1 - e^{-5}\right] \Rightarrow q = CE.[1 - 0.0067]$$

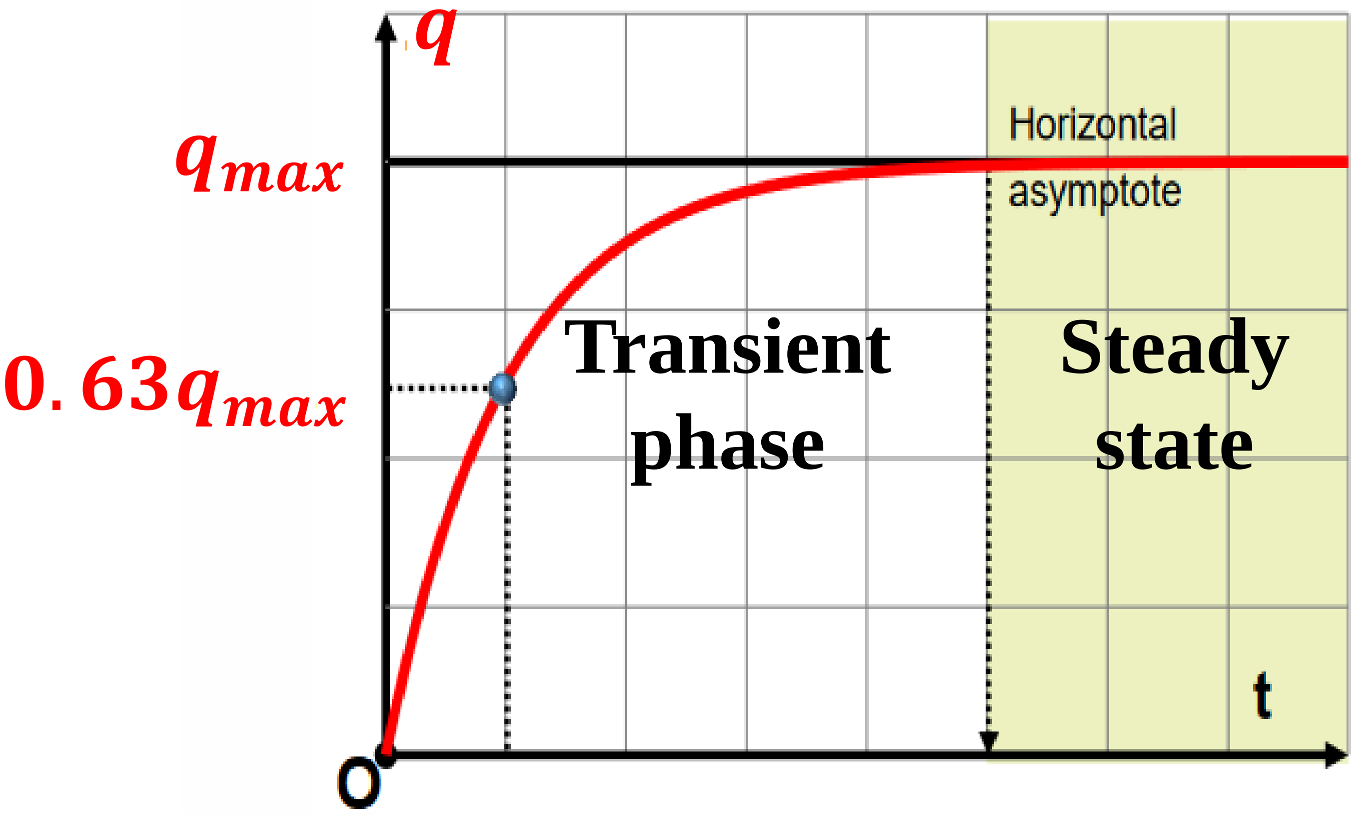
$$q = 0.99CE$$

$$q \approx CE$$

At $t = 5\tau$, the capacitor is practically completely charged.

Summary

$t = 0$	$t = \tau$	$t = 5\tau$
$q = 0$	$q = 0.63q_{max}$	$q = q_{max}$



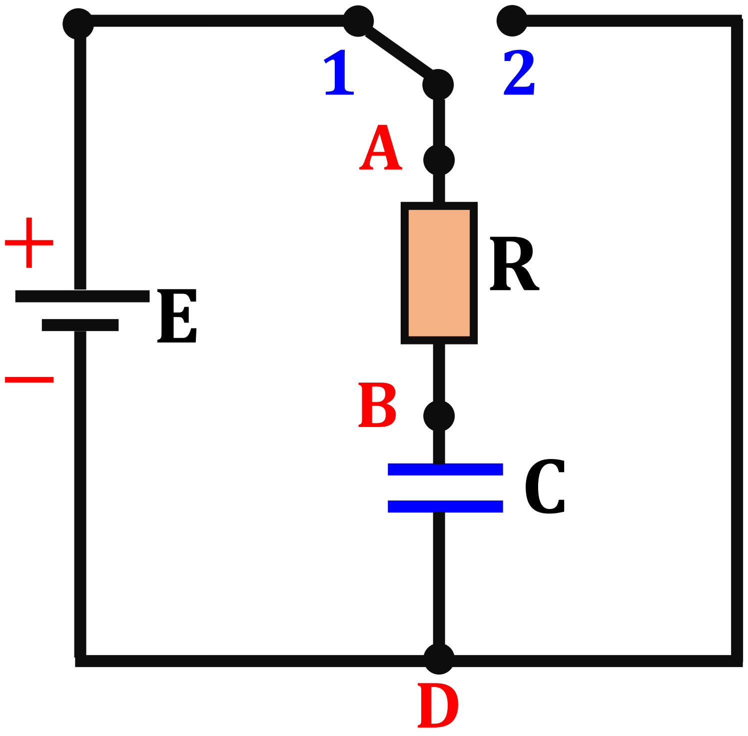
Where

$q_{max} = C.E$

Study of charging of a capacitor theoretically

Application 4:

Consider the electric circuit that consists of an ideal generator delivering, a constant voltage E, a capacitor of capacitance C, a resistor of resistance R and of a switch K.



The capacitor is initially neutral.

At time $t_0 = 0$, we place K at position (1); the capacitor charging phenomenon begins.

Study of charging of a capacitor theoretically

1. Establish the differential equation that describes the variation of q.

2. Show that $q = CE(1 - e^{-\frac{t}{RC}})$ is a solution of the differential equation.

3. The solution of the equation differential in q is of the form $q = A + Be^{-\frac{t}{\tau}}$. Determine A and B in terms of E and C, and τ as a function of RC

Study of charging of a capacitor theoretically

1.Establish the differential equation that describes the variation of the charge q, with respect to time.

Using law of addition of voltages in series:

$$u_G = U_C + U_R$$

$$E = u_C + Ri$$

but $i = +\frac{dq}{dt}$

And $q = Cu_C$ ⇒ $u_C = \frac{q}{C}$

$$E = \frac{q}{C} + R\frac{dq}{dt}$$

$$E.C = q + RC.\frac{dq}{dt}$$

Study of charging of a capacitor theoretically

2.Show that $q = CE(1 - e^{-\frac{t}{RC}})$ is a solution of the differential equation in q.

$$\frac{dq}{dt} = +\frac{CE}{RC}.e^{-\frac{t}{RC}} \Rightarrow \frac{dq}{dt} = \frac{E}{R}.e^{-\frac{t}{RC}}$$

Substitute $\frac{dq}{dt}$ and q in differential equation $C.E = q + RC\frac{dq}{dt}$

$$C.E = CE\left(1 - e^{-\frac{t}{RC}}\right) + RC.\frac{E}{R}.e^{-\frac{t}{RC}}$$

$$C.E = CE - CE.e^{-\frac{t}{RC}} + CE.e^{-\frac{t}{RC}}$$

Study of charging of a capacitor theoretically

$$C.E = CE - CE.e^{-\frac{t}{RC}} + CE.e^{-\frac{t}{RC}}$$

$$C.E = CE$$

Then $q = CE(1 - e^{-\frac{t}{RC}})$ is the solution of the differential equation

Study of charging of a capacitor theoretically

3.The solution of the equation differential is of the form $q = A + Be^{\alpha.t}$. Determine A, B and α in terms of E, R and C.

$$q = A + Be^{\alpha.t}$$

$$\frac{dq}{dt} = \alpha.B.e^{\alpha.t}$$

Substitute $\frac{dq}{dt}$ and q in differential equation

$$C.E = q + RC\frac{dq}{dt}$$

$$C.E = A + Be^{-\frac{t}{\tau}} + RC.\alpha.B.e^{\alpha.t}$$

$$0 = A - C.E + Be^{-\frac{t}{\tau}} + RC.\alpha.B.e^{\alpha.t}$$

$$0 = (A - CE) + Be^{-\frac{t}{\tau}}[1 + RC.\alpha]$$

$$0 = A - CE$$

$$A = CE$$

Study of charging of a capacitor theoretically

$$0 = (A - CE) + Be^{-\frac{t}{\tau}}[1 + RC.\alpha]$$

$$1 + RC.\alpha = 0$$

$$1 = -RC.\alpha$$

$$\alpha = -\frac{1}{RC}$$

$$q = A + Be^{\alpha.t}$$

$$q = C.E + Be^{-\frac{t}{RC}}$$

At $t = 0; q = 0;$

$$q = C.E + Be^{-\frac{t}{RC}}$$

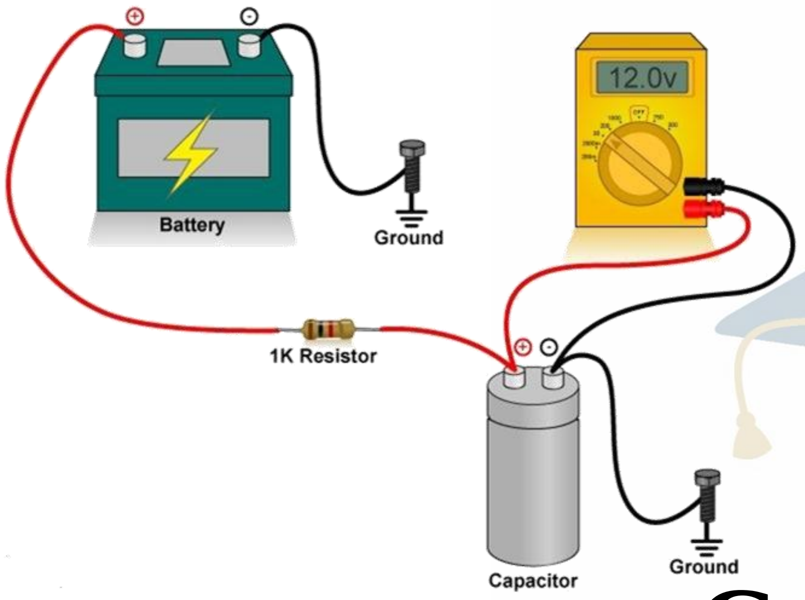
$$0 = CE + Be^{-\frac{0}{RC}}$$

$$0 = CE + Be^{0} \Rightarrow 0 = CE + B$$

$$B = -CE$$

$$q = CE - CEe^{-\frac{t}{RC}}$$

The End



Grade 12 LS – Physics

Chapter 10 -A

Capacitor with a L.F.G of square signal

Prepared & Presented by: Mr. Mohamad Seif

OBJECTIVES

1. Study the differential equation in i in charging phase.
2. Study the solution of the differential equation in i.
3. Calculate the time constant τ in different methods
4. Determine the expression of i and u_R

Study of charging of a capacitor theoretically

Determine the differential equation that governs the variation of i.

Using law of addition of voltages: $u_G = u_C + u_R$

$$E = u_C + Ri$$

Derive w.r.t time:

$$0 = \frac{du_C}{dt} + R\frac{di}{dt}$$

$$i = \frac{dq}{dt}$$ And $$q = Cu_C$$

$$i = C\frac{du_C}{dt} \Rightarrow \frac{i}{C} = \frac{du_C}{dt}$$

$$0 = \frac{i}{C} + R\frac{di}{dt}$$

Study of charging of a capacitor theoretically

The solution of the differential equation in terms of i is:

$$i = I_m e^{-\frac{t}{\tau}}$$

At $t = 0$:

$$i = I_m e^{-\frac{0}{\tau}} \Rightarrow i = I_m e^0 \Rightarrow i = I_m$$

Where $I_{max} = \frac{E}{R}$

Study of charging of a capacitor theoretically

$$i = I_m e^{-\frac{t}{\tau}}$$

At $t = \tau$:

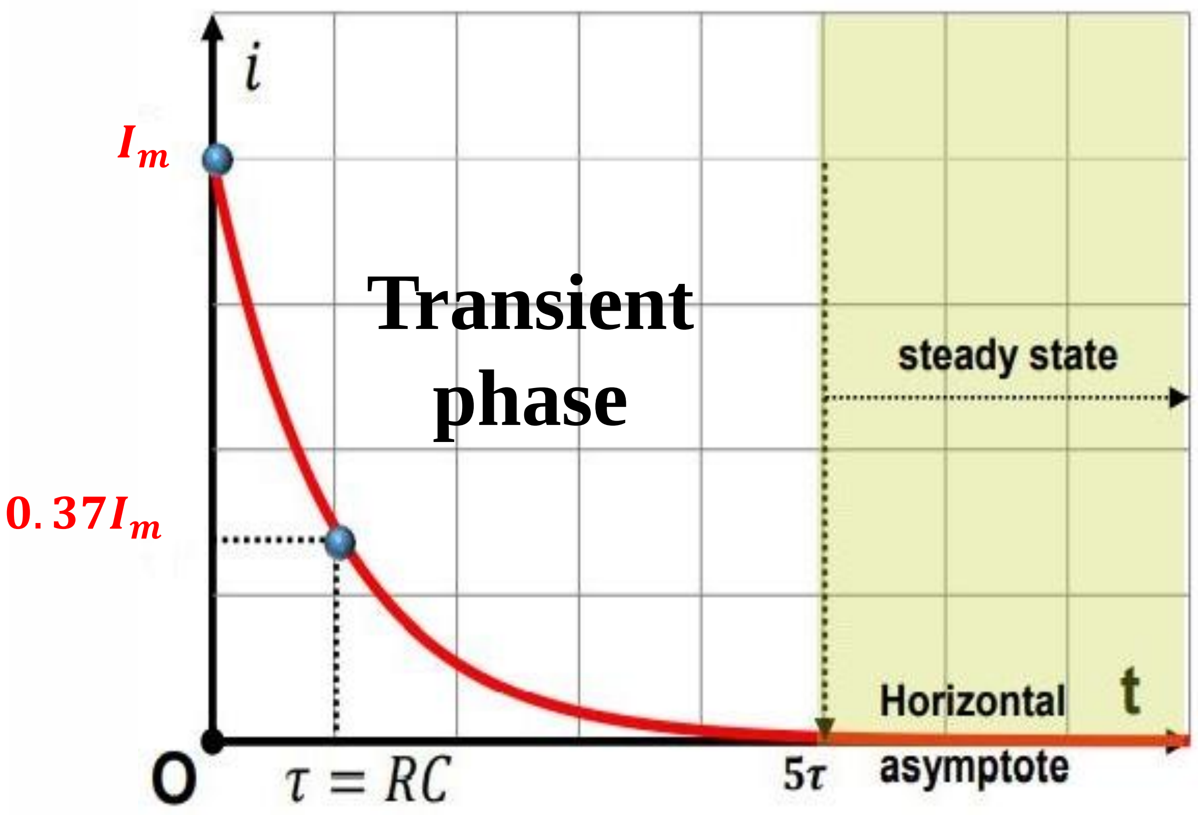
$$i = I_m e^{-\frac{\tau}{\tau}} \Rightarrow i = I_m e^{-1}$$

$$i = 0.37 I_m$$

$t = \tau$: is the time needed for the current in the capacitor loses 63% of its maximum value (I_m)

Study of charging of a capacitor theoretically

Summary		
$t = 0$	$t = \tau$	$t = 5\tau$
$i = I_{max}$	$i = 0.37 I_{max}$	$i = 0$



Where $I_{max} = \frac{E}{R}$

Summary of charging process

t(s)	$t = 0$	$t = \tau = RC$	$t = 5\tau$
u_C	0	$0.63E$	$0.99E \approx E$
q	0	$0.63q_{max}$	$\approx q_{max}$
u_R	E	$0.37E$	0
i	$I_{max} = \frac{E}{R}$	$0.37I_{max}$	0

Study the time constant τ

Application 5:

Consider a circuit consists of a resistor of resistance R=100Ω and a capacitor of capacitance C=200μF connected in series with a generator of voltage $u_g = E$.

Using the equation of time constant: $\tau = RC$

$$\tau = 100 \times (200 \times 10^{-6})$$

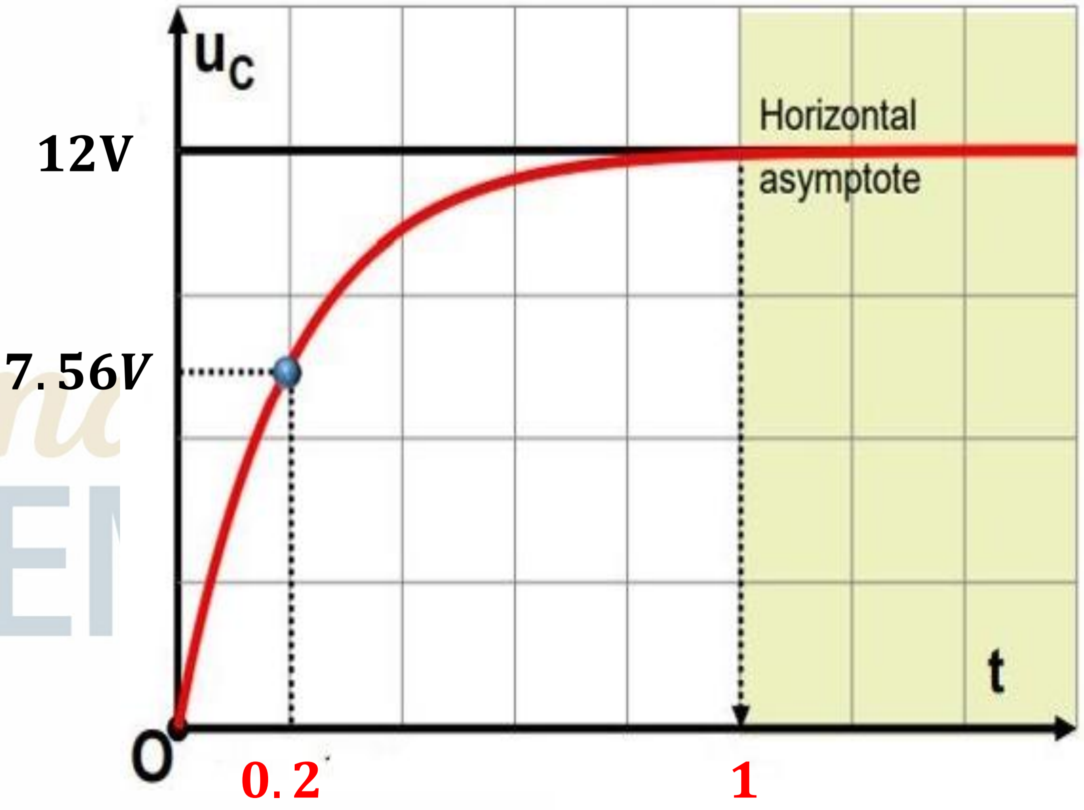
$$\tau = 0.02s$$

Study the time constant τ

Application 6:

Given: resistor of resistance R=5Ω and a capacitor of capacitance C connected in series with generator of voltage $E = 12V$.

Determine the value of time constant τ. Deduce C.



Study the time constant τ

$R=5\Omega$; $u_g = 12V$

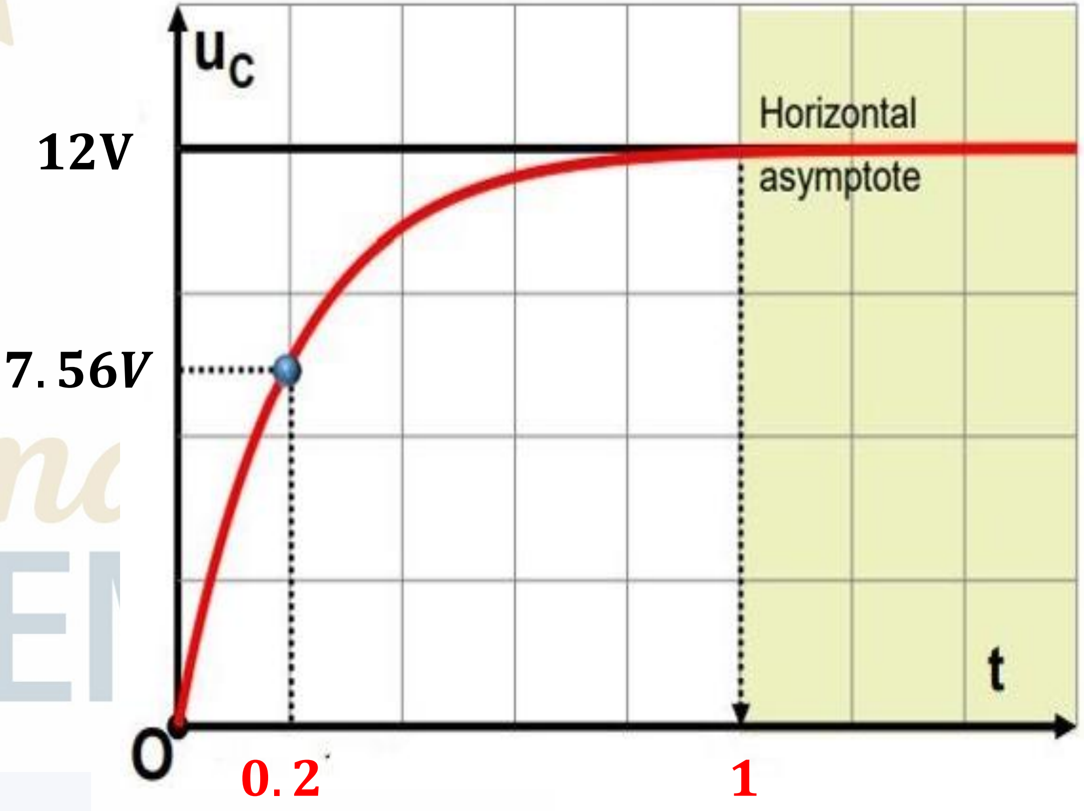
τ: is the time needed for the capacitor to reach 63% of the maximum voltage:

$u_C = 0.63E$ ➔ $u_C = 0.63 \times 12$

$u_C = 7.56V$ ➔ $\tau = 0.2s$

$\tau = RC$ ➔ $C = \frac{\tau}{R}$

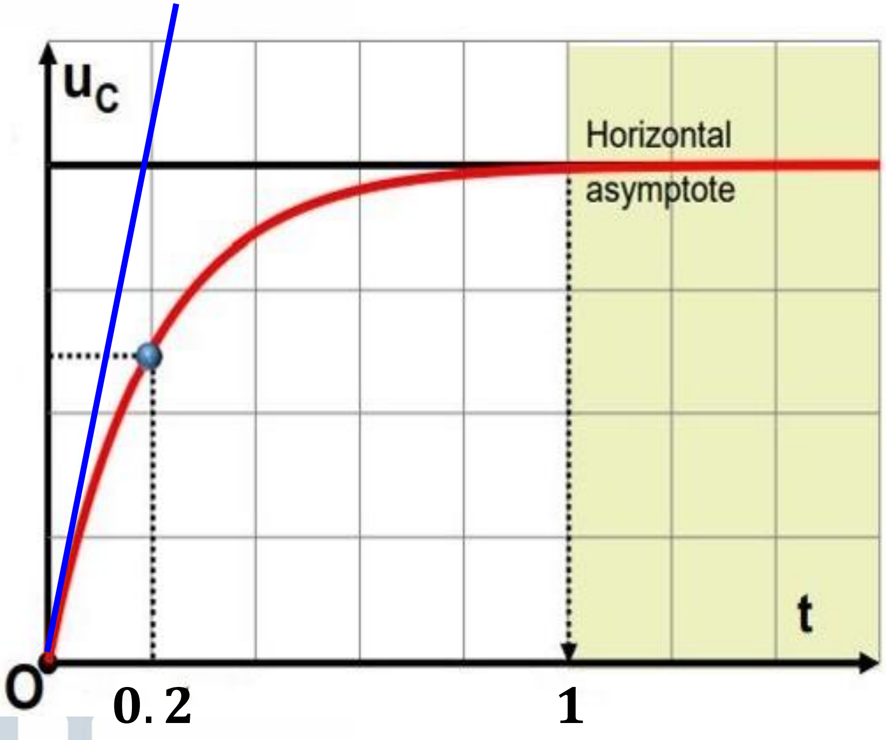
$C = \frac{0.2}{5}$ ➔ $C = 0.04F$



Study the time constant τ

Application 7:

Given: R=5Ω; capacitance C; E = $12V$. Determine the value of time constant τ. Deduce C.



We draw the tangent to the curve of at $t_0 = 0$

The abscissa of the point of intersection between tangent and E is τ: $\tau = 0.2s$

$$C = \frac{\tau}{R} = \frac{0.2}{5}$$

$$C = 0.04F$$

Expression of i and u_R

Given the expression of $u_C = E(1 - e^{-\frac{t}{\tau}})$

$$u_C = E(1 - e^{-\frac{t}{\tau}}) \quad \Rightarrow \quad u_C = E - E.e^{-\frac{t}{\tau}}$$

$$\frac{du_C}{dt} = \frac{E}{\tau}.e^{-\frac{t}{\tau}}$$

$$i = C\frac{du_C}{dt} \quad \Rightarrow \quad i = C.\frac{E}{\tau}.e^{-\frac{t}{\tau}} \quad \Rightarrow \quad i = C.\frac{E}{RC}.e^{-\frac{t}{\tau}}$$

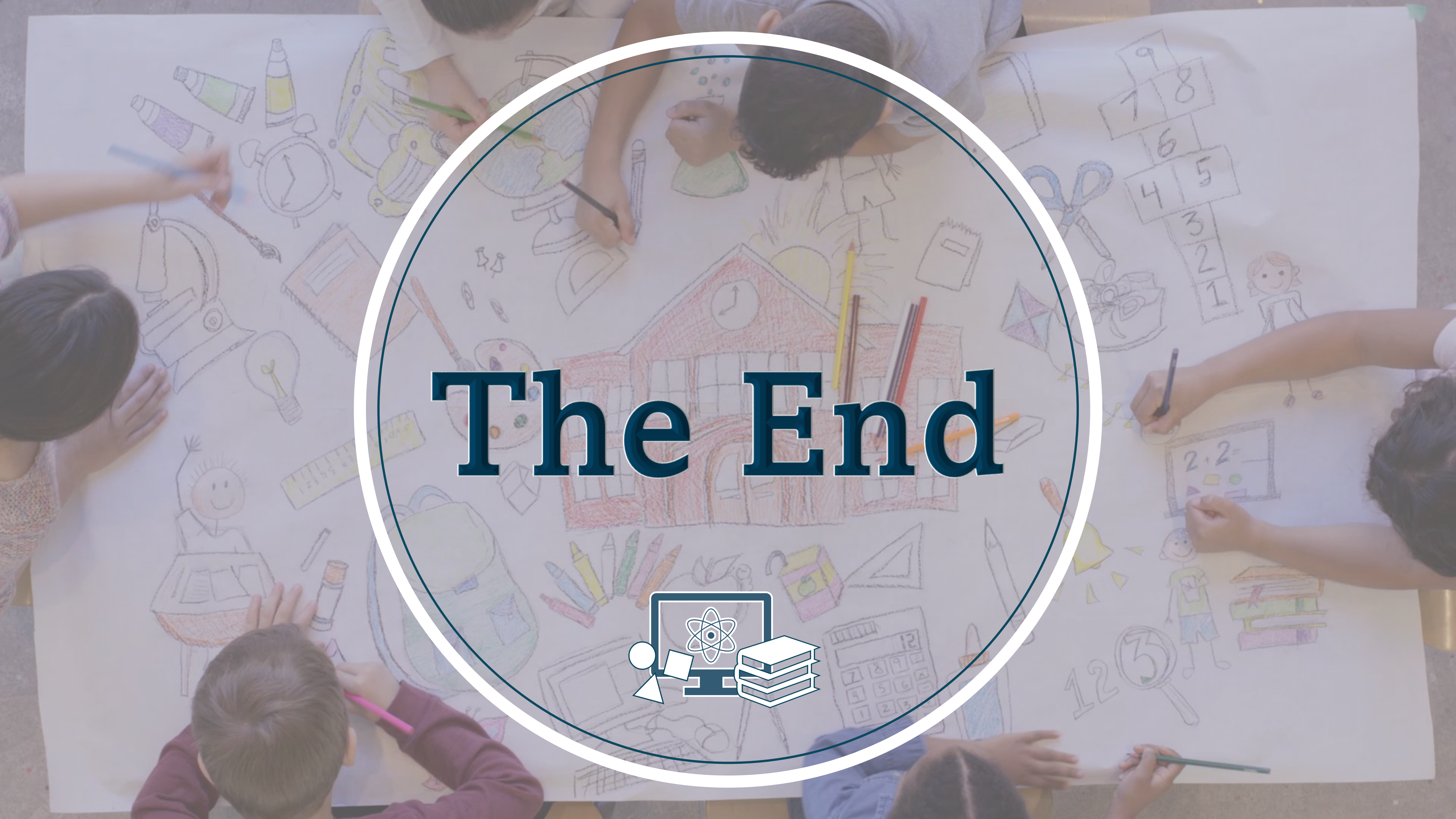
$$i = \frac{E}{R}.e^{-\frac{t}{\tau}}$$

Expression of i and u_R

$$i = \frac{E}{R} . e^{-\frac{t}{\tau}}$$

$$u_R = R \times i \quad \Rightarrow \quad u_R = R \times \frac{E}{R} . e^{-\frac{t}{\tau}}$$

$$u_R = E . e^{-\frac{t}{\tau}}$$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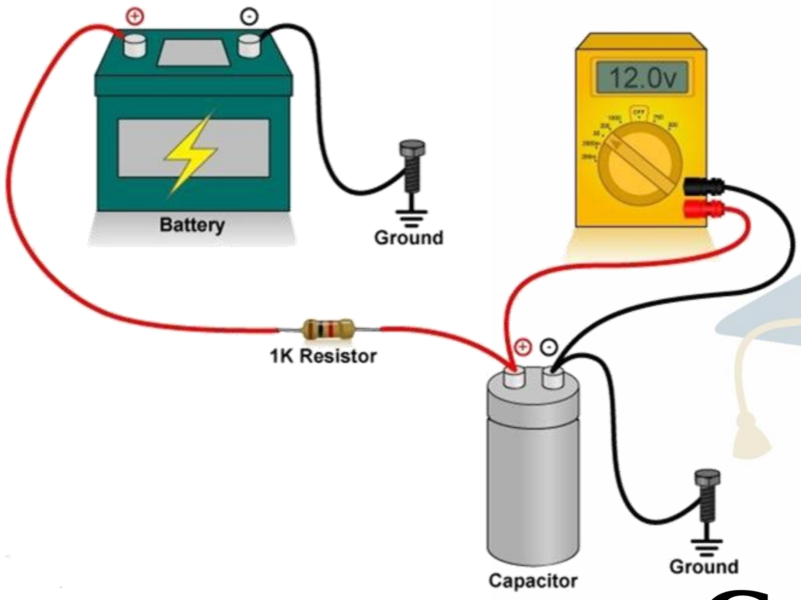


Grade 12 LS – Physics

Chapter 10 -A

Capacitor with a L.F.G of square signal

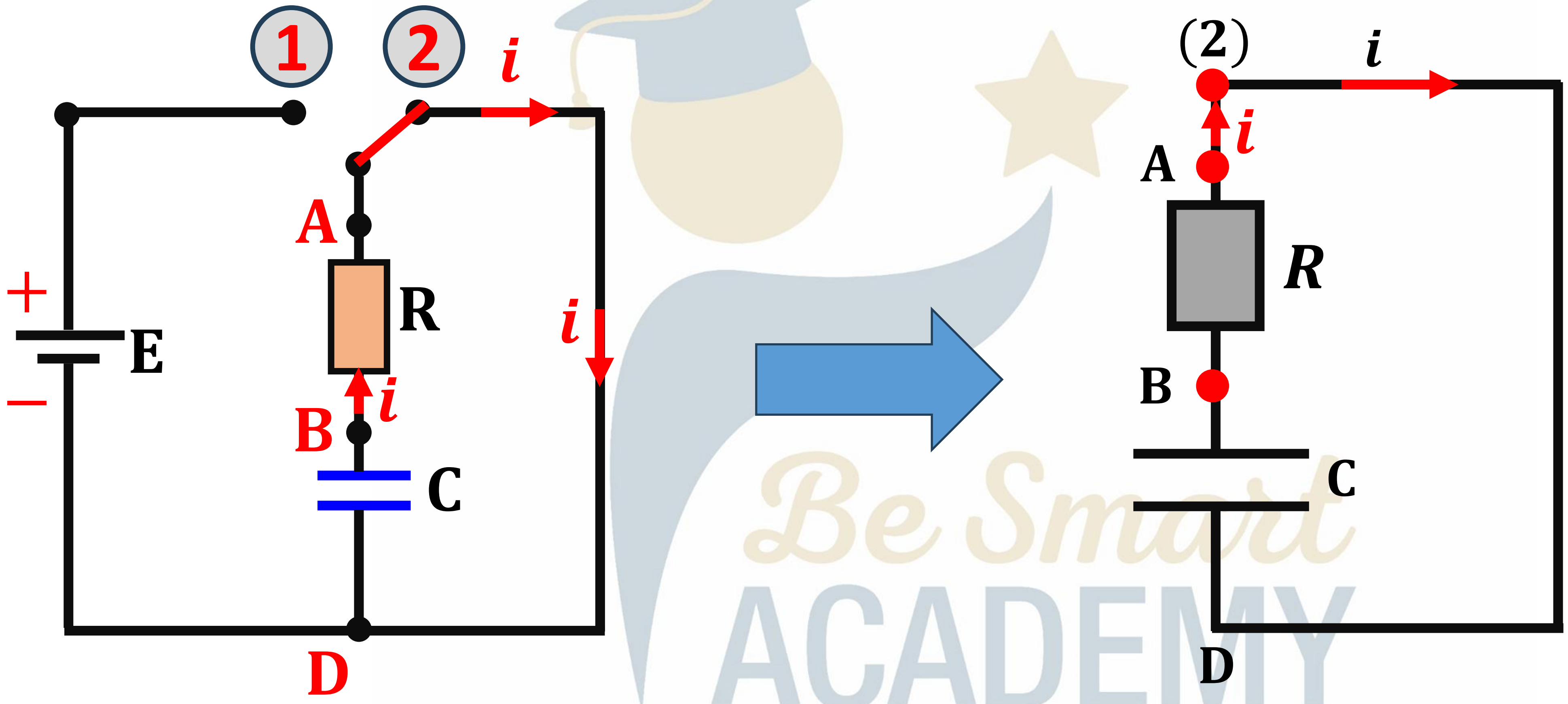
Prepared & Presented by: Mr. Mohamad Seif

OBJECTIVES

1. Study the discharging of a capacitor **experimentally**
2. Study the discharging of a capacitor **theoretically**

Study the discharging of a capacitor experimentally

The switch is turned to position (2), then:



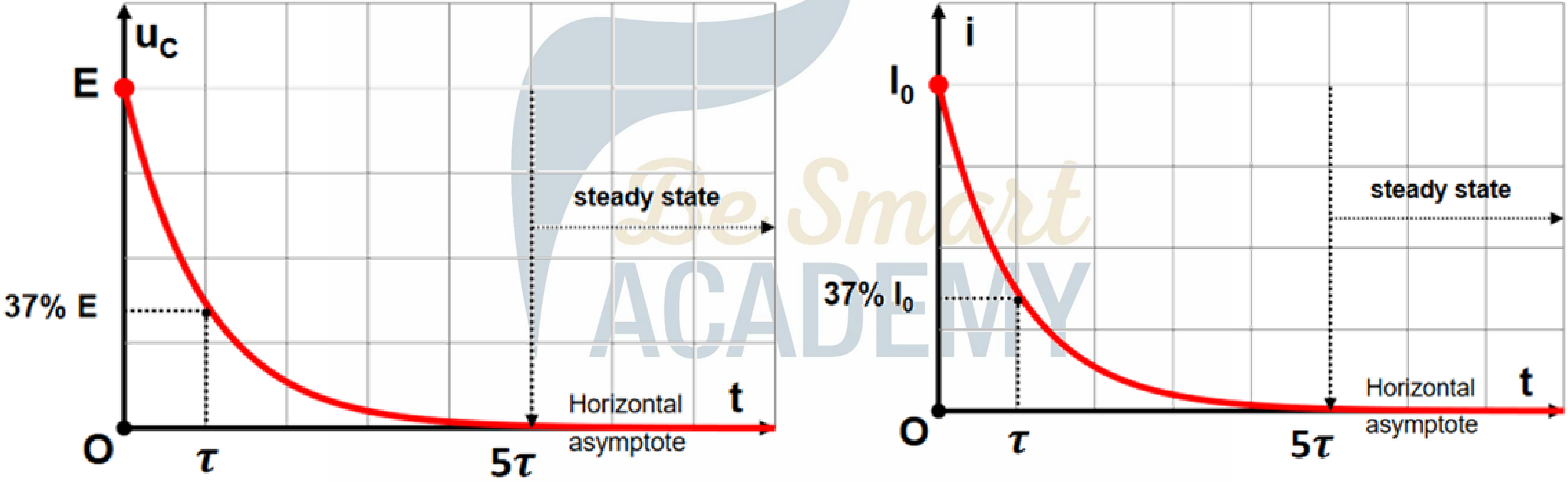
The current leaves the capacitor from the positive plate (B).

The discharging prosses starts.

Study the discharging of a capacitor experimentally

For the switch at point (2): $u_G = 0$ $\quad \frac{T}{2} \leq t \leq T$:

The generator turns off and the following curves are observed on the screen of the oscilloscope.



Study the discharging process of a capacitor theoretically

The first order differential equation of discharging in u_C.

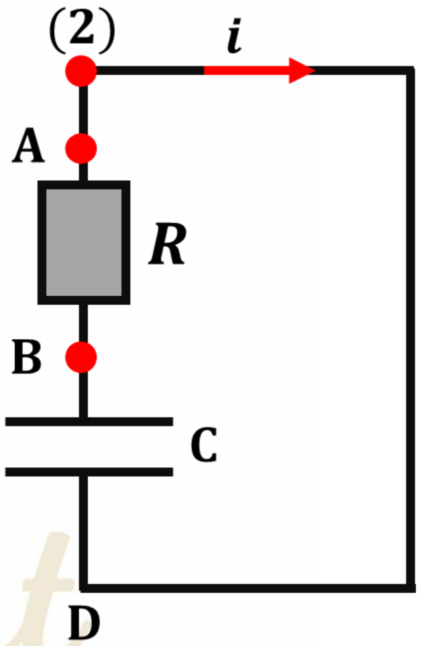
$$u_C = u_R$$

$$u_C = \mathrm{R}i$$

But $i = -\frac{dq}{dt}$ *and* $\mathrm{q} = Cu_C$

$$i = -C\frac{du_C}{dt}$$

$$u_C = -RC\frac{du_C}{dt}$$



$$u_C + RC\frac{du_C}{dt} = 0$$

Study the discharging process of a capacitor **theoretically**

The solution of the differential equation in terms of u_C is:

$$u_C = Ee^{-\frac{t}{RC}}$$

At t = 0:

$$u_C = Ee^{-\frac{0}{\tau}} \Rightarrow u_C = Ee^{0}$$

$$u_C = E$$

Study the discharging process of a capacitor theoretically

$$u_C = Ee^{-\frac{t}{RC}}$$

At t = τ:

$$u_C = Ee^{-\frac{\tau}{\tau}} \Rightarrow u_C = Ee^{-1}$$

$$u_C = 0.37E \qquad u_C = 37\%E$$

$t = \tau$: is the time needed to discharge the capacitor 63% out of the maximum voltage (E).

Study the discharging process of a capacitor theoretically

$$u_C = Ee^{-\frac{t}{RC}}$$

At $t = 5\tau$:

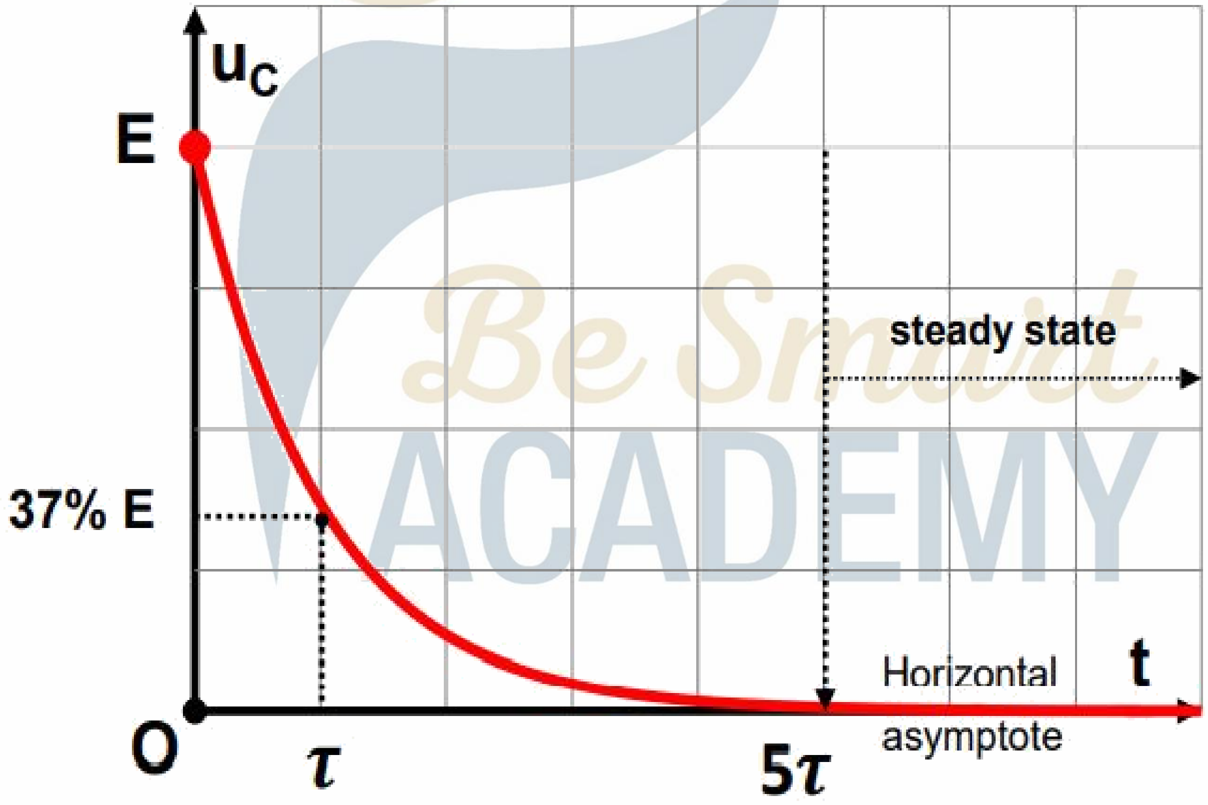
$$u_C = Ee^{-\frac{5\tau}{\tau}} \Rightarrow u_C = Ee^{-5}$$

$$u_C \approx 0$$

At $t = 5\tau$ the capacitor is practically completely discharged

Study the discharging process of a capacitor theoretically

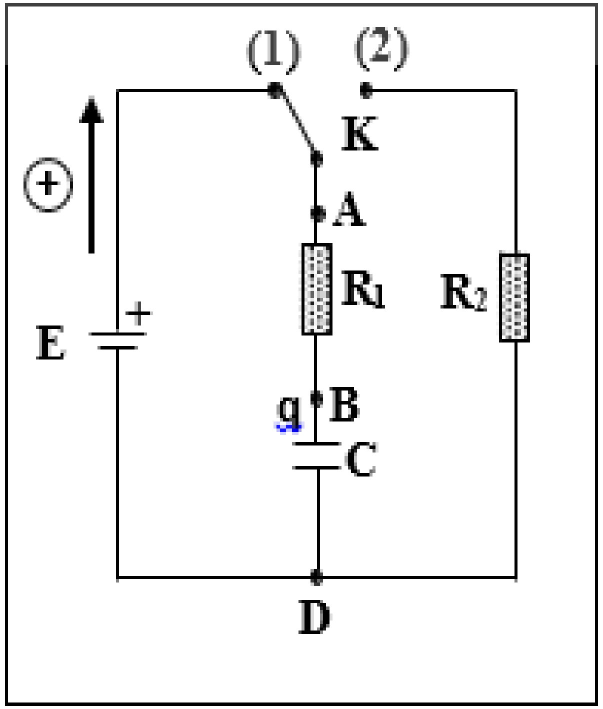
Summary		
$t = 0$	$t = \tau$	$t = 5\tau$
$u_C = E$	$u_C = 0.37.E$	$u_C = 0$



Study the discharging process of a capacitor theoretically

Application 8:

Consider a capacitor initially charged. At a date chosen as a new origin of time (t = 0), we place K at position (2); the capacitor discharge phenomenon begins. At $t = 0$, $u_C = E$.



1. Represent the direction of the current on the circuit.
2. Determine the relationship between i , C and u_C.

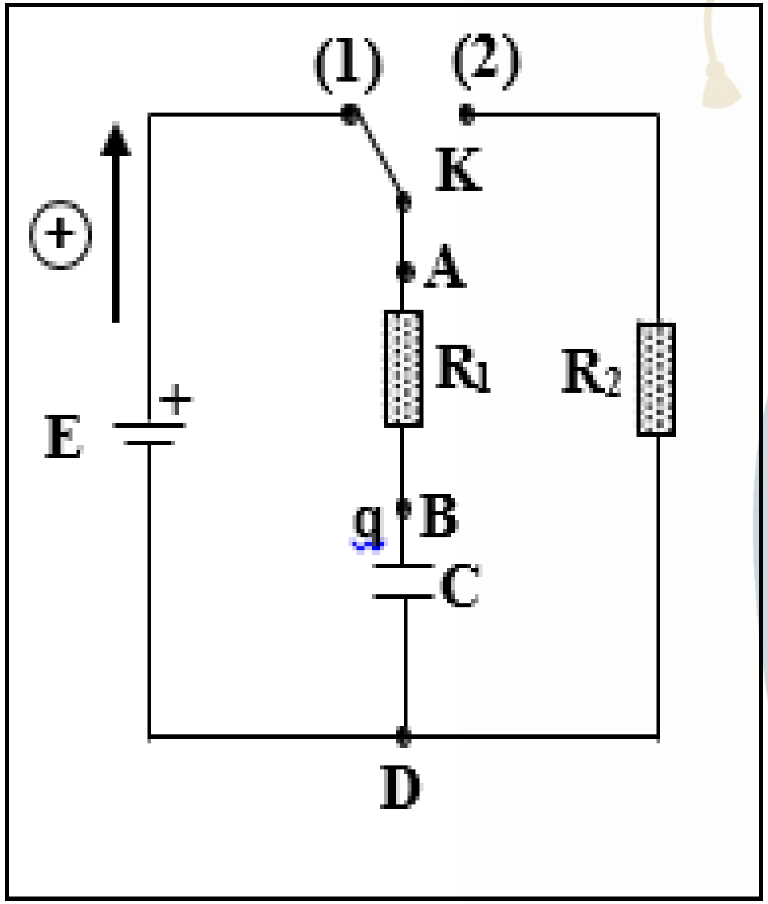
Study the discharging process of a capacitor theoretically

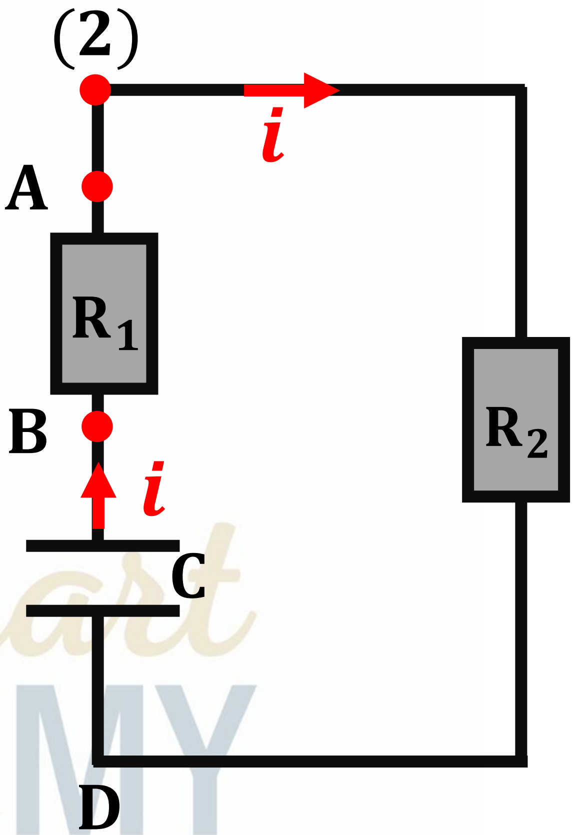
3. Show that the differential equation in terms of u_C, is written in the form: $u_C + RC\frac{du_C}{dt} = 0$.

4. The solution of the above differential equation in u_C is of the form: $u_C = Ae^{-\frac{t}{\tau}}$ Determine A as a function of E, and τ as a function of R and C.

Study the discharging process of a capacitor theoretically

1.Represent the direction of the current on the circuit.





Study the discharging process of a capacitor theoretically

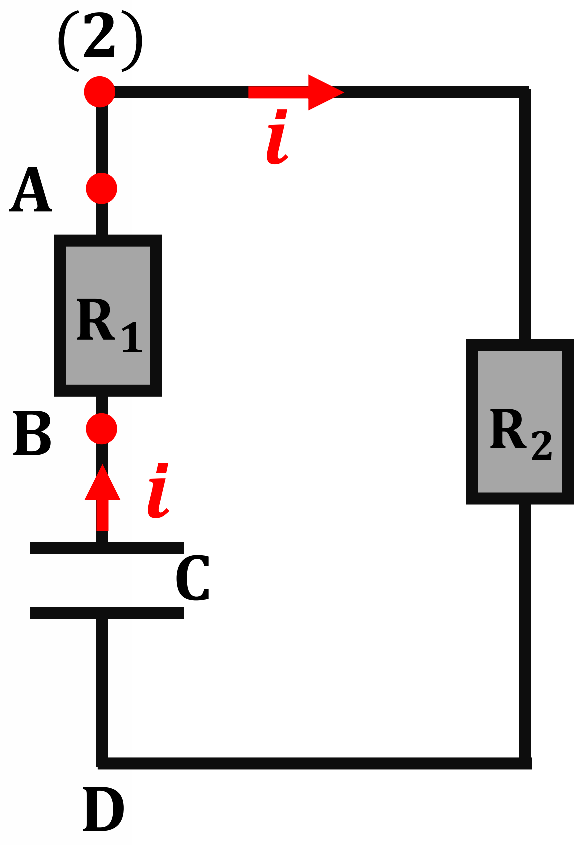
2. Determine the relationship between i, C and u_C.

The capacitor loses charges

$$i = -\frac{dq}{dt}$$

And

$$q = C.u_C$$



$$i = -C.\frac{du_C}{dt}$$

Study the discharging process of a capacitor theoretically

3. Show that the differential equation in terms of u_C, is written in the form: $u_C + \mathrm{RC}\frac{du_C}{dt} = 0$

$$u_C = u_{R1} + u_{R2}$$

$$u_C = R_1 i + R_2 i$$

$$u_C = (R_1 + R_2)i$$

Where $\mathrm{R} = (R_1 + R_2)$

$$u_C = \mathrm{R}i$$

But $i = -\frac{dq}{dt}$ and $\mathrm{q} = \mathrm{C}u_C$

$$i = -C\frac{du_C}{dt}$$

$$u_C = -RC\frac{du_C}{dt}$$

$$u_C + RC\frac{du_C}{dt} = 0$$

Study the discharging process of a capacitor theoretically

4.The solution of the differential equation in u_C is $u_C = Ae^{-\frac{t}{\tau}}$. Determine A and τ as a function of E, R and C.

$$u_C = Ae^{-\frac{t}{\tau}} \Rightarrow \frac{du_C}{dt} = -\frac{A}{\tau}.e^{-\frac{t}{\tau}}$$

Substitute u_C and $\frac{du_C}{dt}$ in differential equation

$$u_C + RC\frac{du_C}{dt} = 0$$

$$A.e^{-\frac{t}{\tau}} - RC.\frac{A}{\tau}.e^{-\frac{t}{\tau}} = 0$$

$$A.e^{-\frac{t}{\tau}}\left(1 - \frac{RC}{\tau}\right) = 0$$

$$1 - \frac{RC}{\tau} = 0 \Rightarrow 1 = \frac{RC}{\tau}$$

$$\tau = RC$$

Study the discharging process of a capacitor theoretically

$$u_C = Ae^{-\frac{t}{RC}}$$

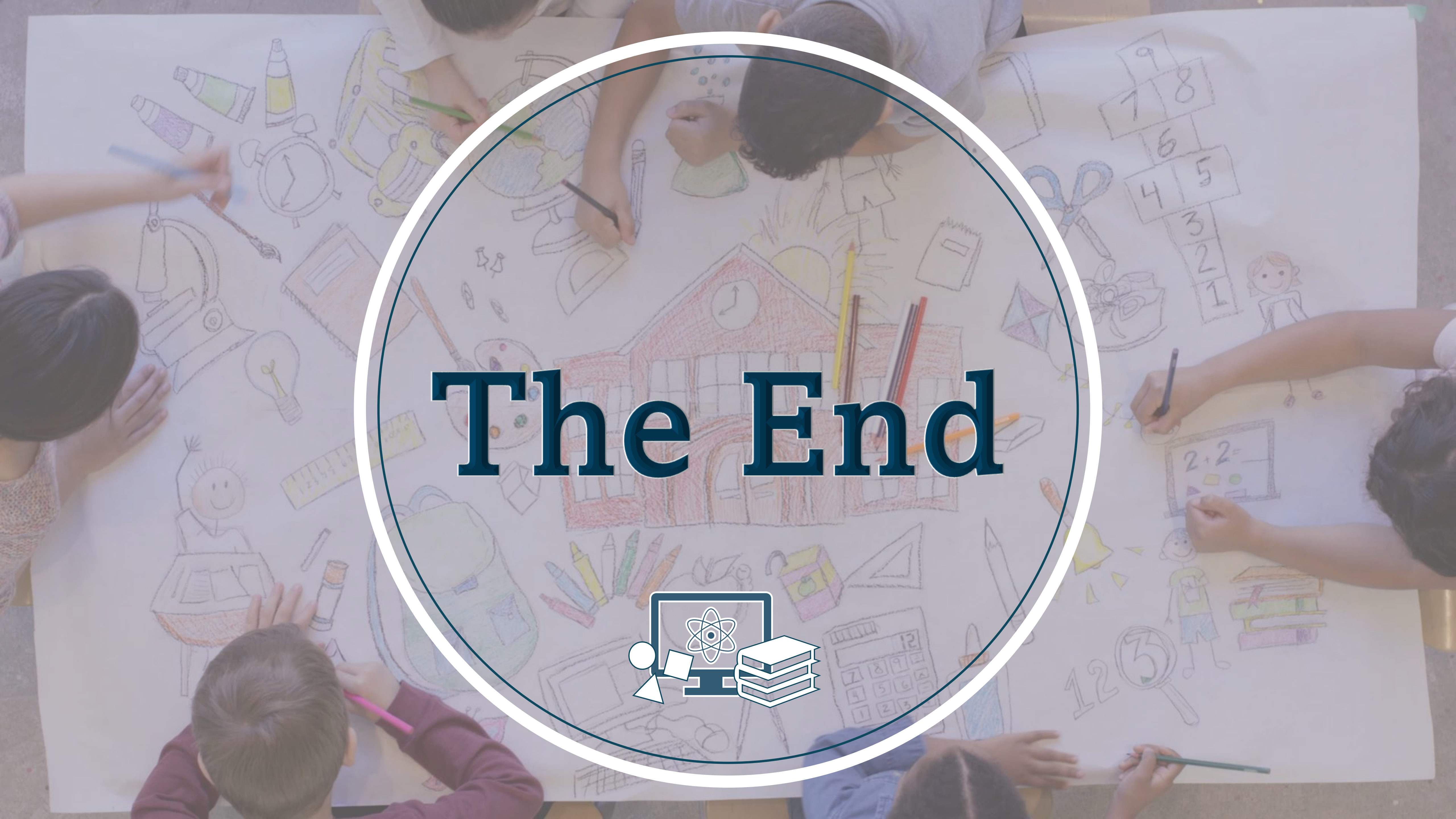
At $t = 0, u_C = E$

$$E = A.e^{-\frac{0}{RC}}$$

$$E = A.e^{0}$$

$$E = A$$

$$u_C = E.e^{-\frac{t}{RC}}$$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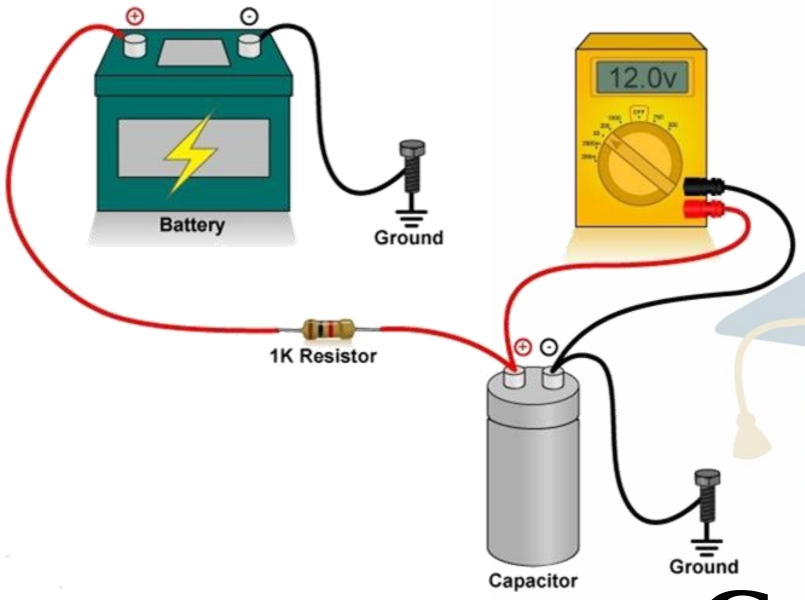


Grade 12 LS – Physics

Chapter 10 -A

Capacitor with a L.F.G of square signal

Prepared & Presented by: Mr. Mohamad Seif

OBJECTIVES

1. Differential equation in terms of q and its solution, during the discharging of a capacitor.
2. Differential equation in terms of i and its solution, during the discharging of a capacitor.

Study the discharging process of a capacitor theoretically

The first order differential equation in terms of q.

$$u_C = u_R$$

$$q = C.u_C \Rightarrow u_C = \frac{q}{C}$$

$$i = -\frac{dq}{dt}$$

$$\frac{q}{C} = -R\frac{dq}{dt}$$

$$\frac{q}{C} + R\frac{dq}{dt} = 0$$

$$q + RC\frac{dq}{dt} = 0$$

Study the discharging process of a capacitor theoretically

The solution of the differential equation in terms of q is

$$q = CEe^{-\frac{t}{RC}}$$

At t=0:

$$q = CEe^{-\frac{0}{\tau}}$$

$$q = CEe^{0}$$

$$q = CE(1)$$

$$q_{max} = CE$$

Study the discharging process of a capacitor theoretically

$$q = CEe^{-\frac{t}{RC}}$$

At $t = \tau$

$$q = CEe^{-\frac{\tau}{\tau}}$$

$$q = CEe^{-1}$$

$$q = 0.37CE$$

$t = \tau$: is the time needed to discharge the capacitor 63% of its maximum value ($q_{max} = CE$)

Study the discharging process of a capacitor **theoretically**

$$q = CEe^{-\frac{t}{RC}}$$

At $t = 5\tau$:

$$q = CEe^{-\frac{5\tau}{\tau}}$$

$$q = CEe^{-5}$$

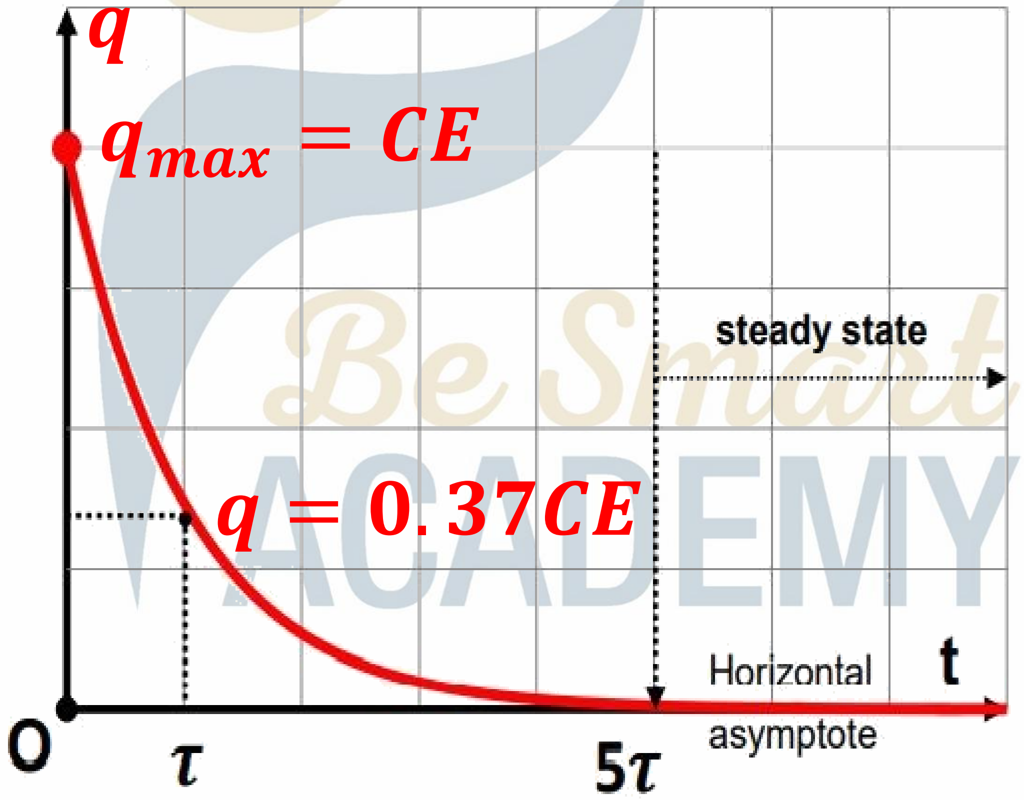
$$q \approx 0$$

At $t = 5\tau$, the capacitor is practically completely discharged

Study the discharging process of a capacitor theoretically

Summary

$t = 0$	$t = \tau$	$t = 5\tau$
$q = CE$	$q = 0.37.CE$	$q = 0$



Study the discharging process of a capacitor theoretically

The differential equation of discharging in terms of i

$$u_C = u_R \Rightarrow u_C - u_R = 0$$

$$u_C - Ri = 0$$

Derive the equation w.r.t time:

$$\frac{du_C}{dt} - R\frac{di}{dt} = 0$$

$$i = -C\frac{du_C}{dt} \Rightarrow \frac{du_C}{dt} = -\frac{i}{C}$$

$$-\frac{i}{C} - R\frac{di}{dt} = 0$$

$$\frac{i}{C} + R\frac{di}{dt} = 0$$

$$\boxed{\frac{di}{dt} + \frac{i}{RC} = 0}$$

Study the discharging process of a capacitor **theoretically**

The solution of the differential equation in terms of i is

$$u = \frac{E}{R} e^{-\frac{t}{RC}}$$

At $t = 0$:

$$i = \frac{E}{R} e^{-\frac{0}{\tau}} \Rightarrow i = \frac{E}{R} e^{0}$$

$$i = \frac{E}{R}(1) \Rightarrow i = \frac{E}{R} = I_{max}$$

Study the discharging process of a capacitor **theoretically**

$$i = \frac{E}{R} e^{-\frac{t}{RC}}$$

At $t = \tau$

$$i = \frac{E}{R} . e^{-\frac{\tau}{\tau}} \Rightarrow i = \frac{E}{R} . e^{-1}$$

$$i = 0.37 . \frac{E}{R}$$

$t = \tau$: is the time needed for the current loses 63% of its maximum value.

Study the discharging process of a capacitor theoretically

$$i = \frac{E}{R} e^{-\frac{t}{RC}}$$

At $t = 5\tau$:

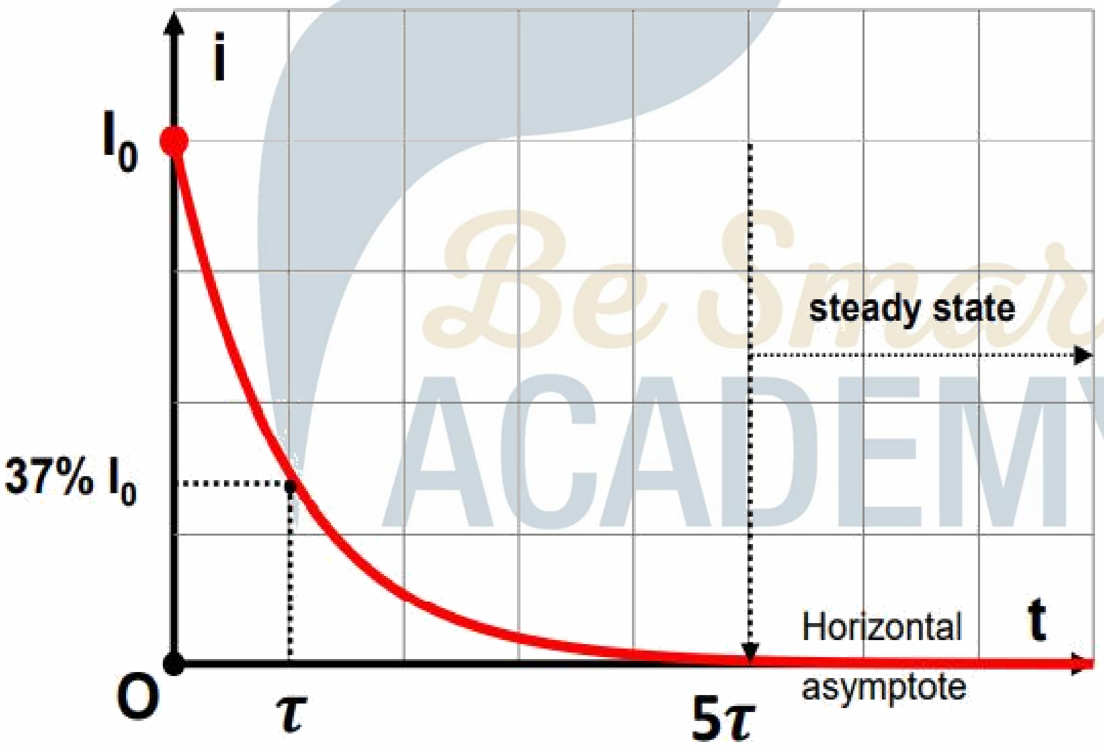
$$i = \frac{E}{R} . e^{-\frac{5\tau}{\tau}}$$

$$i = \frac{E}{R} e^{-5}$$

$$i \approx 0$$

Study the discharging process of a capacitor theoretically

Summary		
$t = 0$	$t = \tau$	$t = 5\tau$
$i = I_{max} = \frac{E}{R}$	$i = 0.37.\frac{E}{R}$	$i = 0$

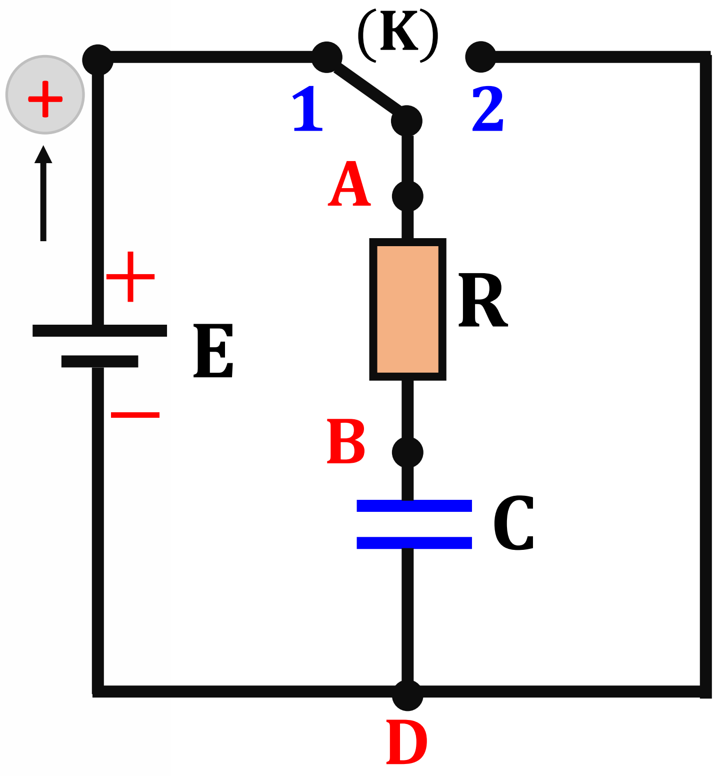


Study the discharging process of a capacitor theoretically

Application 9:

Consider a capacitor initially charged. At a new origin of time (t_0 = 0), we place K at position (2), where u_C = E.

1. Show the direction of current on the circuit.
2. Name the phenomenon takes place.
3. Establish the differential equation that describes the variation of the charge q with respect to at the time.



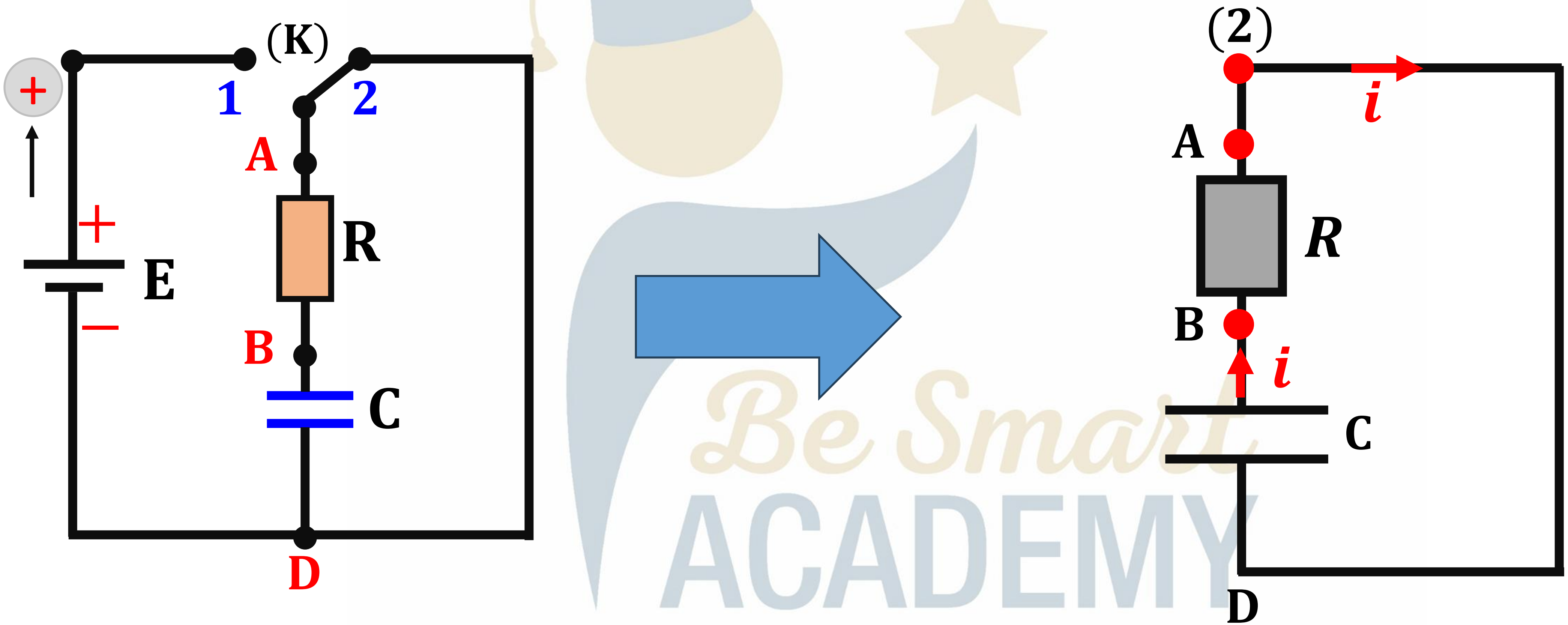
Study the discharging process of a capacitor theoretically

4. The solution of the differential equation in q is of the form $q = Ae^{-\frac{t}{\tau}}$. Determine A as a function of E and C, and τ as a function of RC.

5. Establish the differential equation which describes the variation of the charge i with respect to at the time.

Study the discharging process of a capacitor **theoretically**

1.Show the direction of the current on the circuit.

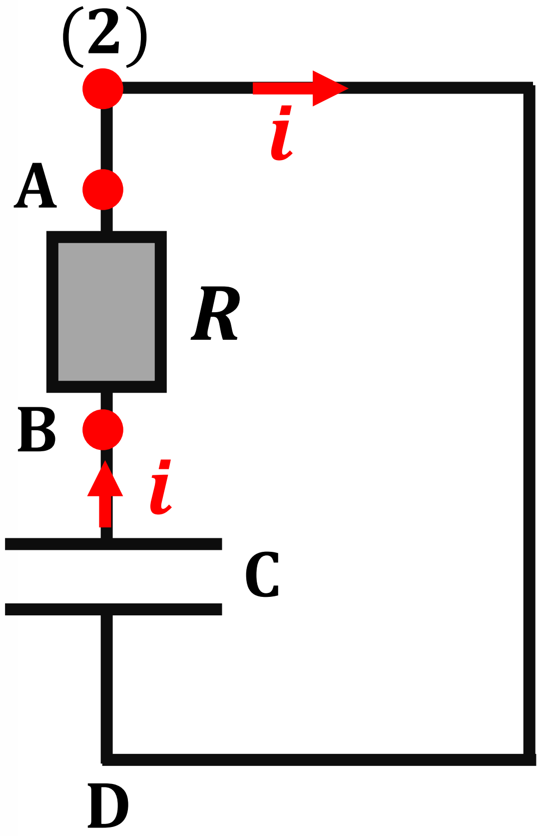


Study the discharging process of a capacitor theoretically

2. Name the phenomenon takes place on the circuit.

When the switch to position (2), the generator is disconnected from the circuit then:

The discharging of capacitor takes place.



Study the discharging process of a capacitor theoretically

3. Establish the differential equation which describes the variation of the charge q with respect to at the time.

$$u_C = u_R \qquad u_C - u_R = 0$$

$$q = C.u_C \Rightarrow u_C = \frac{q}{C}$$

$$\frac{q}{C} - Ri = 0$$

$$i = -\frac{dq}{dt}$$

$$\frac{q}{C} - R\left(-\frac{dq}{dt}\right) = 0$$

$$\frac{q}{C} + R\frac{dq}{dt} = 0$$

$$q + RC\frac{dq}{dt} = 0$$

Study the discharging process of a capacitor theoretically

4. The solution of the differential equation is $q = Ae^{-\frac{t}{\tau}}$. Determine A and τ in terms of E, R and C.

$$q = Ae^{-\frac{t}{\tau}} \Rightarrow \frac{dq}{dt} = -\frac{A}{\tau}.e^{-\frac{t}{\tau}}$$

Substitute q and $\frac{dq}{dt}$ in differential equation

$$q + RC\frac{dq}{dt} = 0$$

$$A.e^{-\frac{t}{\tau}} - RC.\frac{A}{\tau}.e^{-\frac{t}{\tau}} = 0$$

$$A.e^{-\frac{t}{\tau}}\left(1 - \frac{RC}{\tau}\right) = 0$$

$$1 - \frac{RC}{\tau} = 0 \Rightarrow 1 = \frac{RC}{\tau}$$

$$\tau = RC$$

Study the discharging process of a capacitor theoretically

$$q = Ae^{-\frac{t}{RC}}$$

At $t = 0, q = CE$

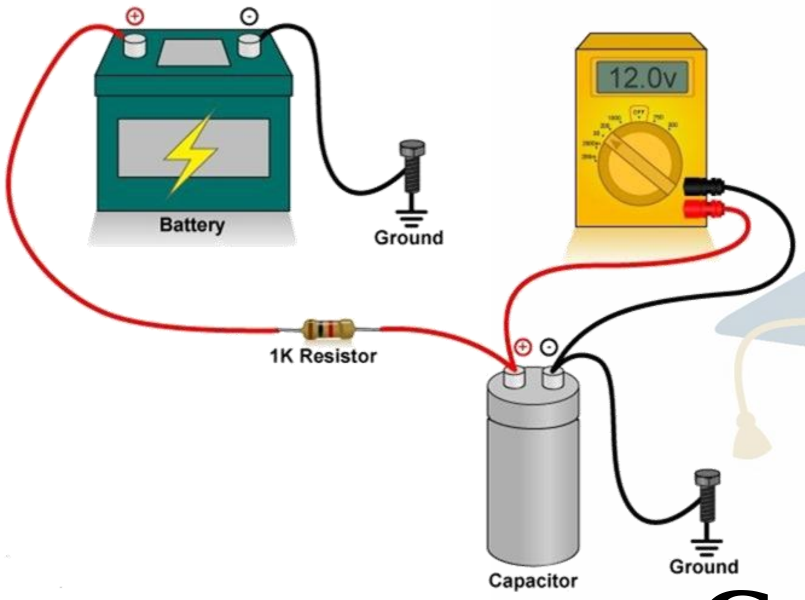
$$CE = A.e^{-\frac{0}{RC}}$$

$$CE = A.e^{0}$$

$$A = CE$$

$$q = CE.e^{-\frac{t}{RC}}$$

The End



Grade 12 LS – Physics

Chapter 10 -A

Capacitor with a L.F.G of square signal

Prepared & Presented by: Mr. Mohamad Seif

OBJECTIVES

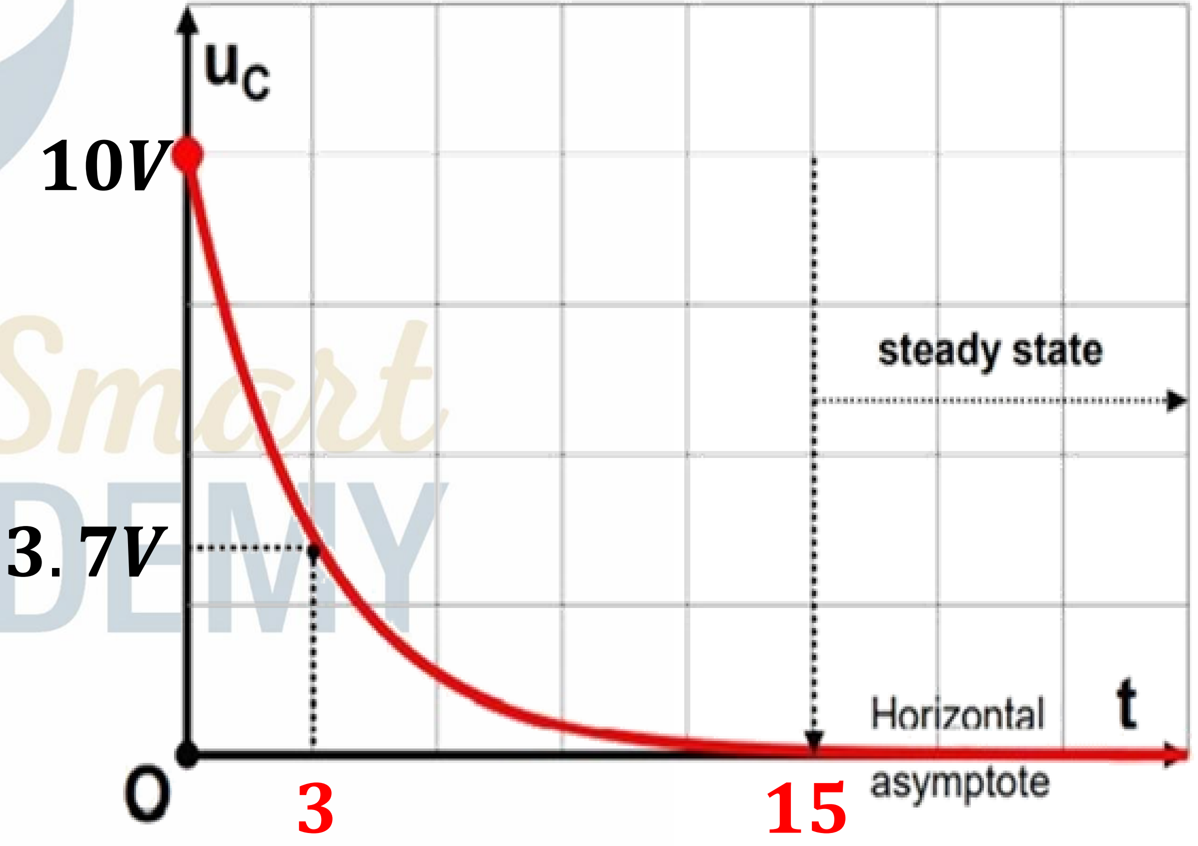
1 To calculate the time constant τ during the discharging

Time constant τ during the discharging

Application 10:

A circuit consists of a resistor of resistance $R = 10K\Omega$, a capacitor of capacitance C and a generator of voltage $E = 10V$.

1. Determine the value of time constant τ.
2. Deduce the value of C.



Calculate the time constant τ during the discharging

$R = 10K\Omega$; $E = 10V$.

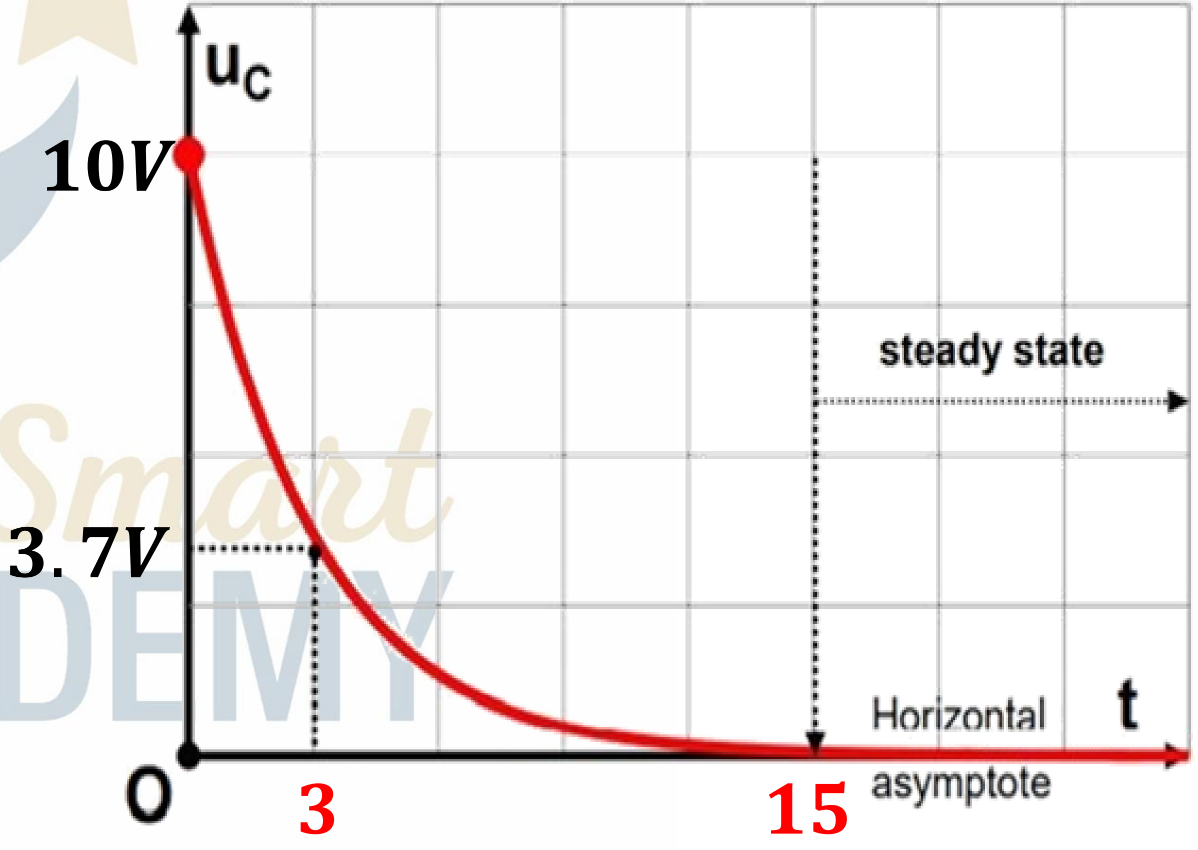
1. Determine the value of time constant τ.

τ: is the time needed for the capacitor to discharge 63% out of the maximum voltage:

$$u_C = 0.37E$$

$$u_C = 0.37 \times 10$$

$u_C = 3.7V \Rightarrow \tau = 3s$



Calculate the time constant τ during the discharging

$R = 10K\Omega$; E = 10V.

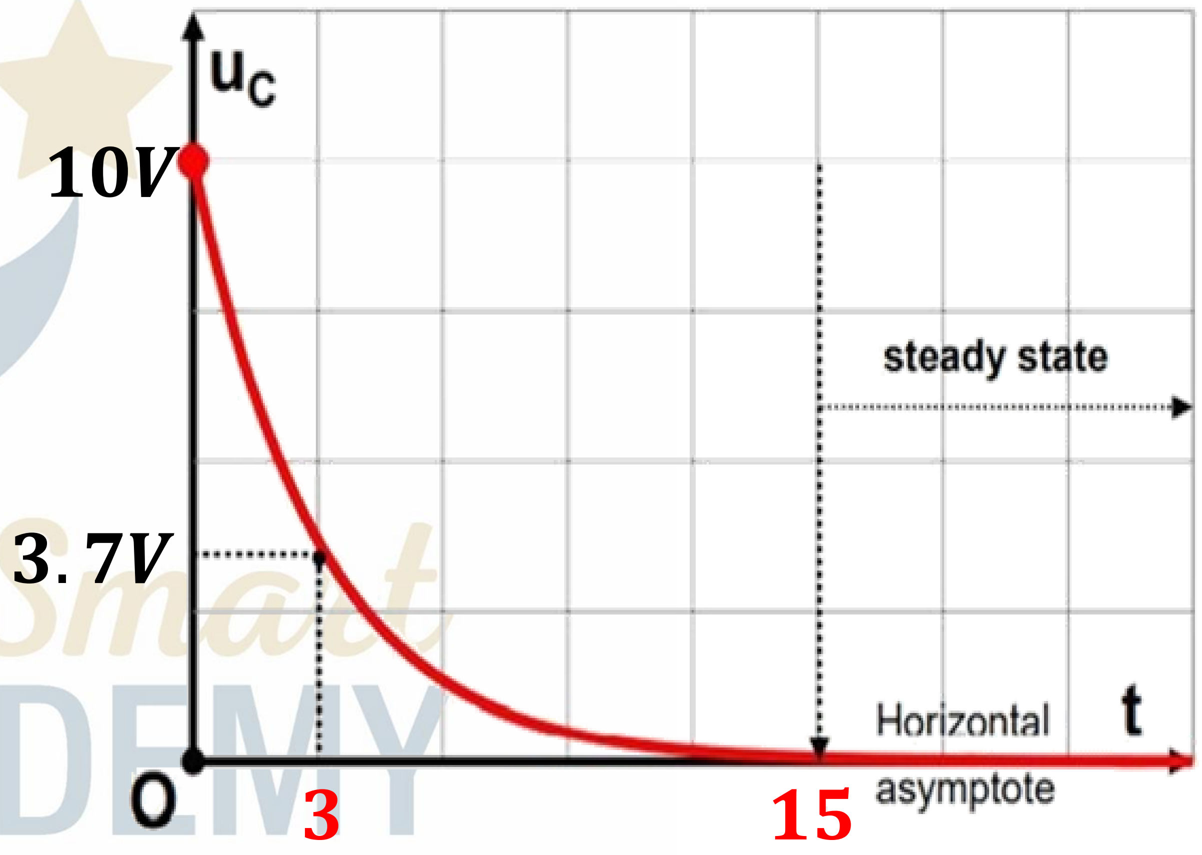
2. Deduce the value of C.

$$\tau = RC$$

$$C = \frac{\tau}{R}$$

$$C = \frac{3}{10000}$$

$$C = 3 \times 10^{-4} F$$

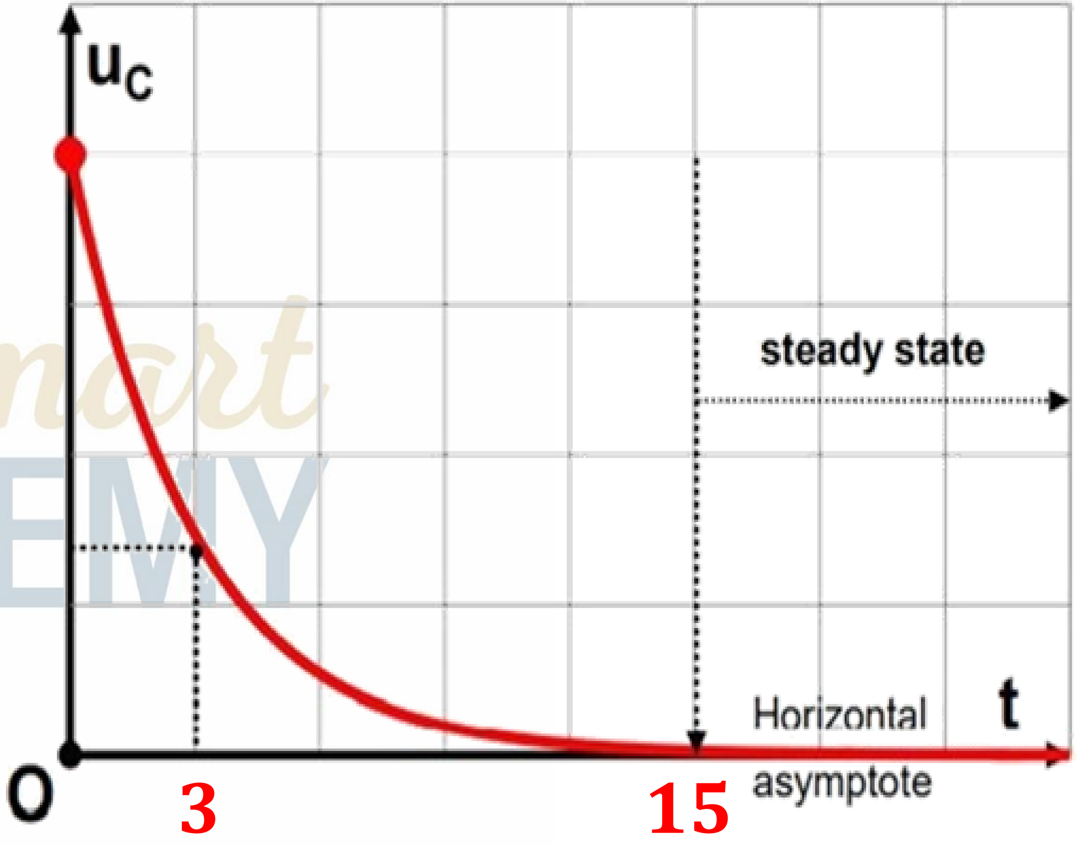


Calculate the time constant τ during the discharging

Application 11:

A circuit consists of a resistor of resistance $R = 10K\Omega$, a capacitor of capacitance C and a generator of voltage E.

1. Determine the value of time constant τ using tangent method.
2. Deduce the value of C.



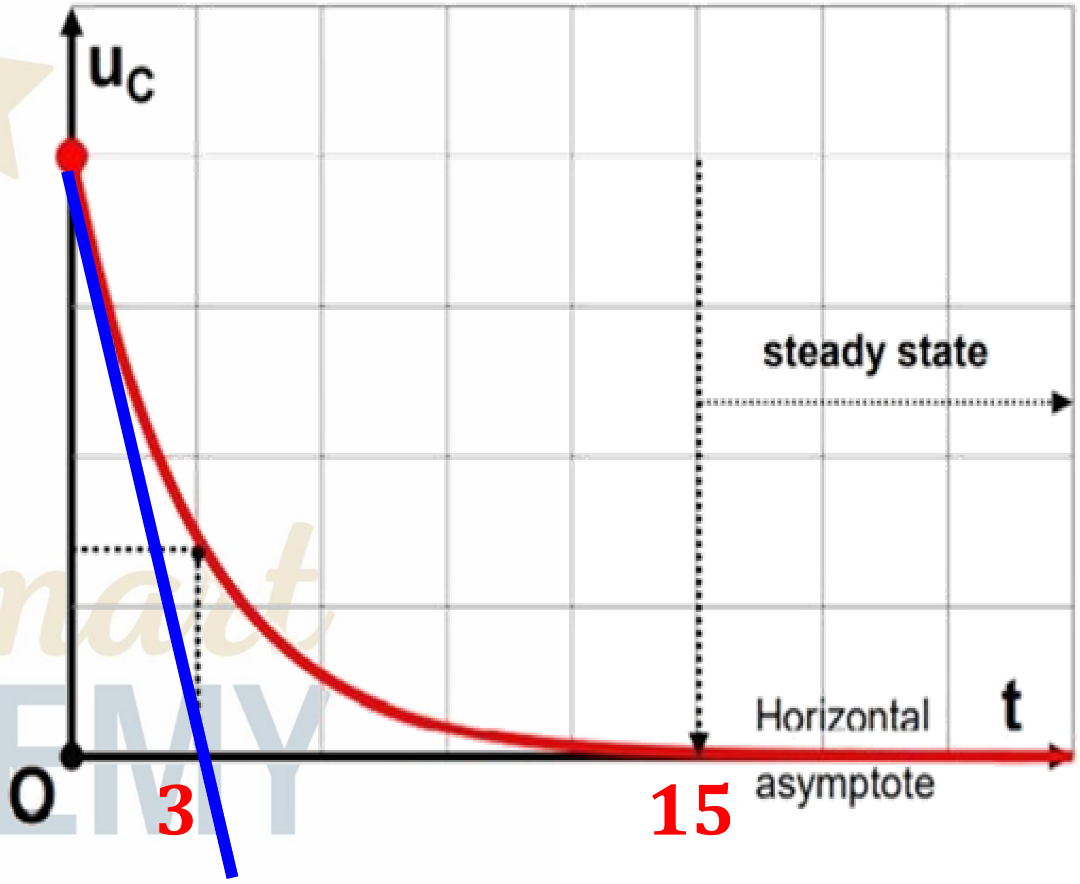
Calculate the time constant τ during the discharging

1. Determine the value of time constant τ using tangent method.

Draw a tangent to curve at $t = 0$

The intersection between tangent and time axis is the time constant

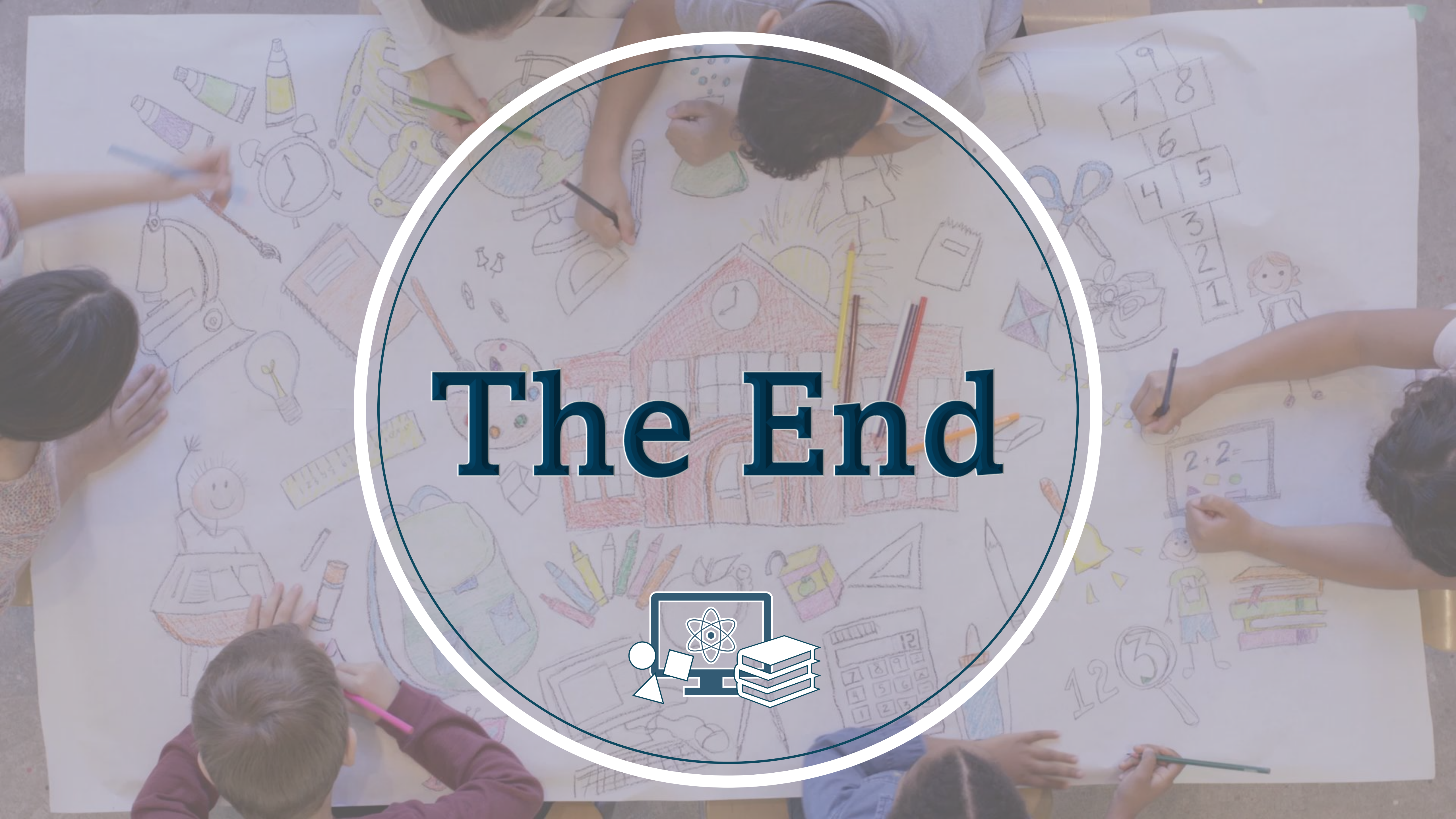
$$\tau = 3s$$



2. Deduce the value of C.

$$\tau = RC \Rightarrow C = \frac{\tau}{R} = \frac{3}{10000}$$

$$C = 3 \times 10^{-4} F$$


The End